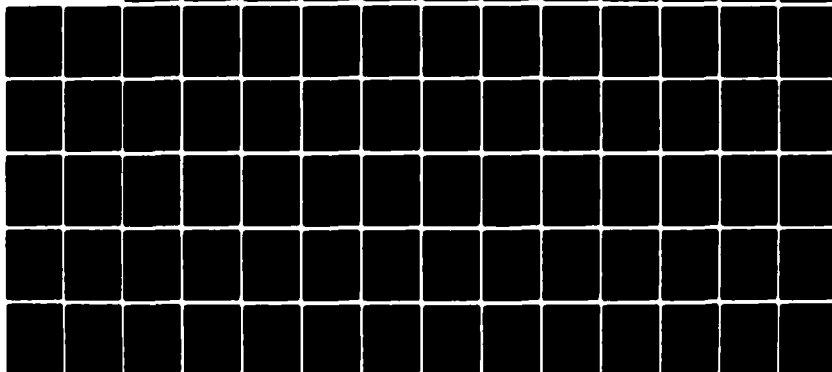


AD-A098 583

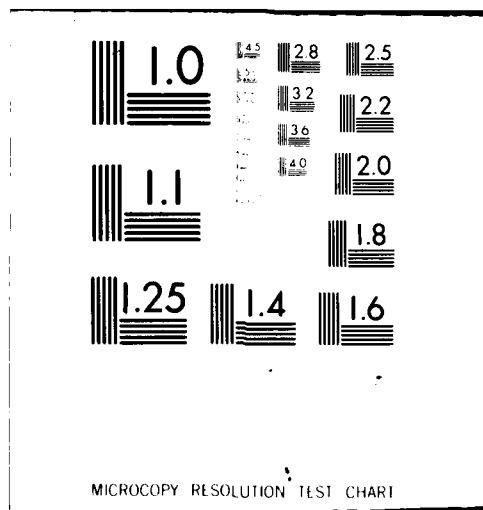
NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/8 10/2  
PERFORMANCE OF AN EHD POWER GENERATOR WITH A TWO-FLUID EJECTOR. (U)  
JAN 81 T H GAWAIN, O BIBLARZ ER-78-A-03-2122  
NPS-80-004 NL

UNCLASSIFIED

1 of 1  
AD-A  
000 000



END  
DATE  
FILMED  
5-81  
DTIC



LEVEL

2

NPS67-80-004

NAVAL POSTGRADUATE SCHOOL  
Monterey, California



DTIC  
ELECTED  
MAY 5 1981  
C

PERFORMANCE OF AN EHD POWER GENERATOR  
WITH A TWO-FLUID EJECTOR

by

T. H. Gawain and O. Biblarz

January 1981

Approved for public release; distribution unlimited.

Prepared for:  
U.S. Department of Energy  
Washington, DC 20545

81 5 04 079

AD A098583

DTIC FILE COPY

NAVAL POSTGRADUATE SCHOOL  
Monterey, California

Rear Admiral J. J. Ekelund  
Superintendent

D. A. Schrady  
Acting Provost

The work reported herein was supported by the U.S. Department of Energy, Washington, D.C. This is the final report on this project.

Reproduction of all or part of this report is authorized.

This report was prepared by:

T. H. Gawain  
T. H. GAWAIN  
Professor of Aeronautics

Oscar Biblarz  
O. BIBLARZ, Associate  
Professor of Aeronautics

Reviewed by:

Released by:

[ M. F. Platzer  
M. F. PLATZER, Chairman  
Department of Aeronautics

William M. Tolles  
W. M. TOLLES  
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
(11) 2 NPS67-80-004	AD-A098	583
4. TITLE (and Subtitle)		5. TYPE OF REPORT & PERIOD COVERED
(6) Performance of an EHD Power Generator with A Two-Fluid Ejector,		
7. AUTHOR(s)		8. CONTRACT OR GRANT NUMBER(s)
(10) T. H./Gawain <del>and</del> O./Biblarz		(15) Interagency Agreement No. ER-78-A-03-2122
9. PERFORMING ORGANIZATION NAME AND ADDRESS		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Naval Postgraduate School Monterey, CA 93940		(13) 861
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE
Department of Energy Division of Advanced Energy Project Washington, DC 20545		Jan 1981
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		13. NUMBER OF PAGES
(9) Final rept.		78
		15. SECURITY CLASS. (of this report)
		UNCLASSIFIED
		15a. DECLASSIFICATION, DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)		
Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)		
(See next page)		

DD FORM 1473  
1 JAN 73EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE/When Data Entered

A detailed analysis and method of calculation is presented for determining the complete thermodynamic cycle of a two-fluid electrohydrodynamic (EHD) power generator. The analysis takes fully into account the compressibility of the media. Parameters are included which express the thermodynamic losses in the various components of the overall system. The severe restriction on output created by the electrical breakdown limit of the medium is clearly shown. The method for computing the net-electrical work output per unit mass of primary fluid and the net overall thermal efficiency of the system is carefully developed. A sample output together with the FORTRAN program are included.

DD Form 1473  
Jan 73  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE/When Data Entered

# ABSTRACT

A detailed analysis and method of calculation is presented for determining the complete thermodynamic cycle of a two-fluid electrohydrodynamic (EHD) power generator. The analysis takes fully into account the compressibility of the media. Parameters are included which express the thermodynamic losses in the various components of the overall system. The severe restriction on output created by the electrical breakdown limit of the medium is clearly shown. The method for computing the net-electrical work output per unit mass of primary fluid and the net overall thermal efficiency of the system is carefully developed. A sample output together with the FORTRAN program are included.

Accession For	
NTIS GRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Avail and/or	
Dist	Special
A	

# TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT . . . . .	iii
1. Background . . . . .	1
2. Introduction . . . . .	5
3. Ejector . . . . .	11
4. Adiabatic Mixing of Two Streams . . . . .	16
5. Mass Flow . . . . .	17
6. Entropy . . . . .	22
7. Momentum . . . . .	26
8. Availability . . . . .	30
9. Diffuser . . . . .	33
10. Electrical Power Section . . . . .	37
11. Condenser/Separator . . . . .	41
12. Cycle Efficiency . . . . .	44
13. Calculation Sequence . . . . .	47
14. Optimum Velocity Ratio . . . . .	56
15. Conclusions and Recommendations . . . . .	58
16. Sample Output . . . . .	60
17. Computer Program . . . . .	63
18. Nomenclature . . . . .	73
19. References . . . . .	76
DISTRIBUTION LIST . . . . .	78



## LIST OF FIGURES

	<u>PAGE</u>
Figure 1.1 Schematic of Power Generator . . .	4
Figure 2.1 Schematic Flow Diagram of Two-Fluid EHD Cycle . . . . .	6
Figure 3.1 Comparison of Actual Ejector with Ideal Ejector and with Hypothetical Minimum Loss Device . . . . .	7
Figure 5.1 Approximate Sketch of Function $f(M)$ .	20
Figure 7.1 Approximate Sketch of Function $g(M)$ .	28

## 1. Background

This is the final report of the project "Study of Mark's Electrothermodynamic (ETD) Power Generator", sponsored by the Department of Energy (DOE). The primary objective of this effort has been to provide a detailed theoretical analysis of a type of electrohydrodynamic (EHD) power generator proposed by the Marks Polarized Corporation. Once a sound analytical basis was developed, it was possible to ascertain the limitations of the proposed system and to seek improvements.

The results of our first study are reported in detail in Ref. (1) and summarized more concisely in Ref. (2). That work indicated the originally proposed design would yield unacceptably poor performance and suggested a certain design revision. Further work was done to analyze such a design revision and is reported in the Appendix to Ref. (1). An improved simplified and shortened version of that work is reported in Ref. (3). Unfortunately, the new results indicated that the revised design, while showing somewhat improved performance, still would not appear sufficiently competitive in overall efficiency.

In the first study, some minor effects of compressibility of the medium were neglected. This was done in order to simplify the calculations. Also, it has been conjectured by Marks and others that it might be possible to improve greatly

the performance of EHD-generators which utilizes an ejector by employing a two-fluid cycle. Finally, performance gains that a diffuser might bring about appeared worthy of study. To this end, a second study for a two-fluid cycle was undertaken in which a diffuser is introduced just prior to the EHD generator and where full account is taken of the compressibility of both fluids at all locations.

The principal earlier effort along the lines of our second study is Ref. (4), by Huberman, et. al, but our present work has gone beyond anything undertaken in Ref. (4). Although a two-fluid generator is analyzed in Ref. (4), including the compressibility of both fluids, Huberman et. al, make no attempt to study the complete thermodynamic cycle. There are, moreover, a number of questionable assumptions which cast serious doubt in the validity of the results given in Ref. (4).

The computational scheme that we have developed was reported in Ref. (5). There a methodical derivation of the working formulas is given together with some preliminary results. Some further results are given in Ref. (6).

The present report updates the material given in Refs (5) and (6). In particular, we have found that instead of writing an energy balance across the condenser, it is more convenient to use a momentum balance as shown in Section II of this report. We have reproduced herein the complete analysis in its latest version.

A sample calculation for the two-fluid system is also included together with the computer program listing.

Figure 1.1 is a schematic of the generator configuration studied. In Marks' design, the centerline of the channel, which is shown in Fig. 1.1 as a straight line, is curved so as to form a closed loop.

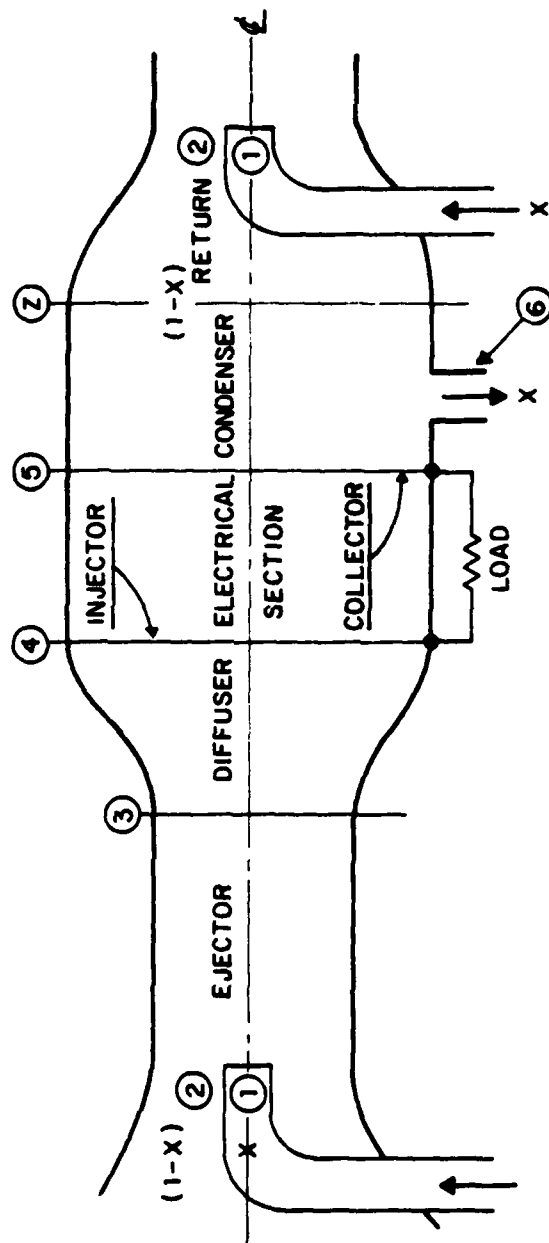


Fig. 1.1 Schematic of Generator Configuration

## 2. Introduction

For purposes of analysis, the overall thermodynamic process in the two-fluid EHD generator may be represented by the schematic flow diagram shown in Fig. 2.1.

Primary fluid enters the ejector at station 1, secondary fluid enters at station 2 and the mixture leaves at station 3.

The mixture then passes through a diffuser, process 3→4, the basic purpose of which is to reduce the kinetic energy of the flow and thereby decrease the associated friction losses downstream of the diffuser. On the other hand the diffuser introduces certain losses of its own so that some care is necessary to optimize the design in this respect.

The gas mixture then passes through the electrical power section, process 4→5, from which the gross electrical power is extracted.

Finally, the gas mixture enters the condenser/separator at station 5. The primary fluid is separated from the gas mixture by condensation and leaves at station 6. The fluid at this point is assumed to be compressed liquid at a known static pressure  $P_6$  and a known static temperature  $T_6$ . Kinetic energy at station 6 is regarded as negligible. The secondary fluid leaves the condenser/separator and enters the ejector at station 2. We assume, moreover, that

$$P_6 = P_2 = P_1 \quad (2.1)$$

and

$$T_6 = T_2 \quad (2.2)$$

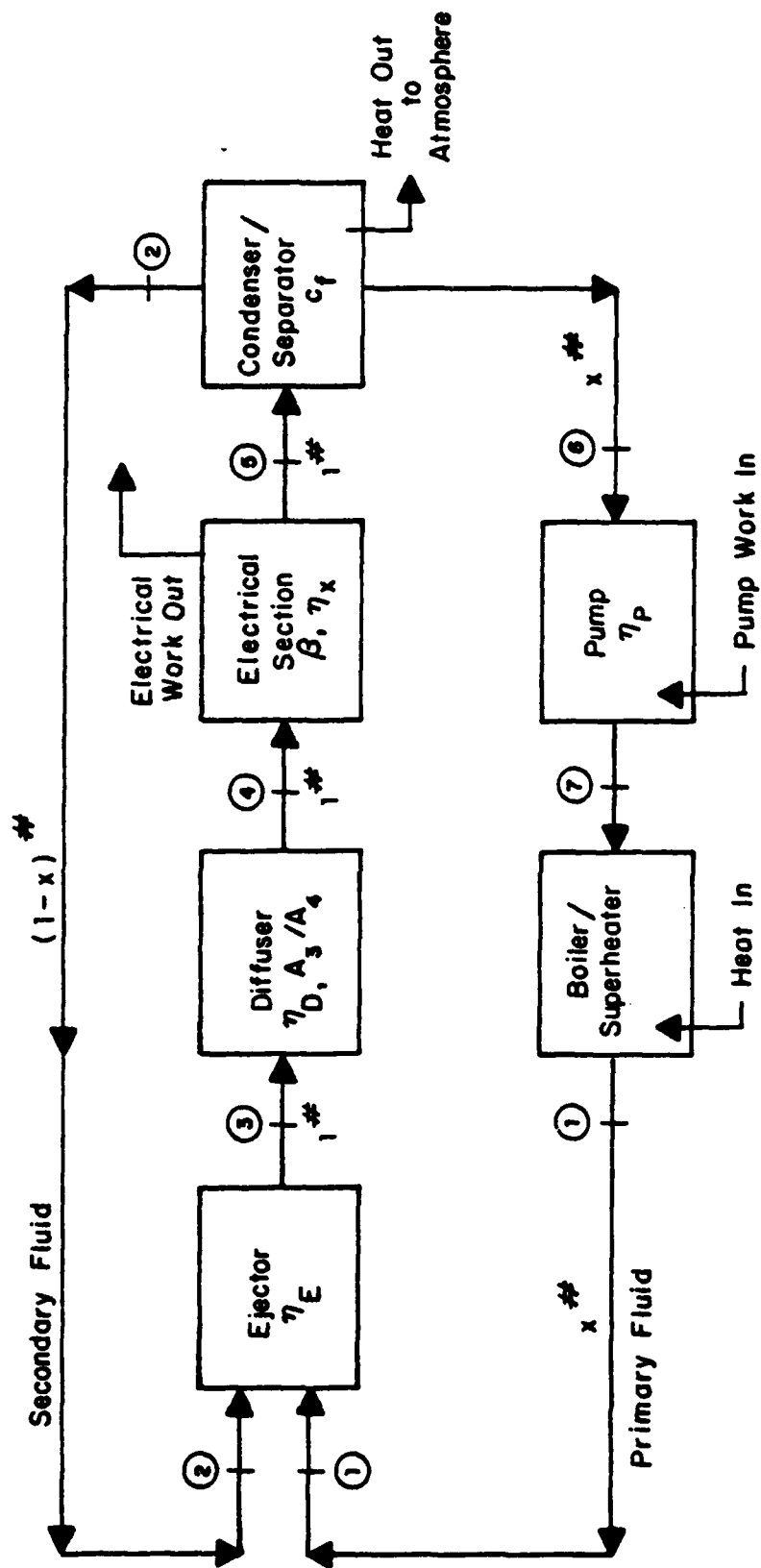


Fig.2.1 SCHEMATIC FLOW DIAGRAM OF TWO - FLUID EHD CYCLE.

The primary fluid which leaves the condenser/separator at station 6 is circulated back through the pump and through the boiler/superheater, process 7+1.

It should also be explained that the primary fluid at station 1 is a condensible vapor of high molecular weight which contains fine liquid droplets. Unless specifically stated otherwise the quality  $z$  at this location is taken as 0.95. The important thermodynamic static properties at station 1 can be expressed in the following form.

$$h_1 = h_f + z h_{fg} \quad \text{Enthalpy} \quad (2.3)$$

$$s_1 = s_f + z s_{fg} \quad \text{Entropy} \quad (2.4)$$

Here subscript  $f$  denotes the saturated liquid and subscript  $fg$  denotes the change between saturated liquid and saturated vapor. All properties are evaluated at a specified pressure  $P_1$ ; the corresponding saturation temperature  $T_1$  is then also known.

The corresponding stagnation properties  $T_{s1}$  and  $h_{s1}$  are fixed when the stagnation pressure  $P_{s1}$  is specified. Of course the entropy has the same value  $s_1$  at the stagnation condition as at the corresponding static state. Thus specification of  $P_{s1}$  and  $s_1$  suffices to fix all other stagnation properties.

While it is convenient to start the analyses by stipulating the quality at station 1, it is also advisable later in the calculation to determine the corresponding quality at



station 4, the entrance to the electrical working section. It is essential that the fluid entering the working section contain finely dispersed liquid droplets of the proper size. These droplets carry the electrical charges that are the central feature of EHD power generation. We assume tentatively that acceptable droplet characteristics are obtained by maintaining the quality at the inlet to the working section in the range 0.92 to 0.98. These limits are only estimates.

The thermodynamic analysis would be greatly simplified if both fluids were perfect gases with constant specific heats. In fact only the secondary fluid satisfies this requirement. Nevertheless, we can define an "equivalent perfect gas" which adequately approximates the essential thermodynamic properties of the primary vapor/liquid mixture. When carried out judiciously, this procedure simplifies the analysis at the cost of only a small loss in numerical accuracy<sup>7</sup>.

The ratio of specific heats  $\gamma$  of the "equivalent perfect gas" may be deduced from the important relation

$$(P_{s1}/P_1) = (T_{s1}/T_1)^{\gamma_1/(\gamma_1 - 1)} \quad (2.5)$$

Solving this for  $\gamma_1$  gives

$$\gamma_1 = \left[ 1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)} \right]^{-1} \quad (2.6)$$

Another fundamental property of the "equivalent perfect gas" is its specific heat  $C_{p1}$  which may be evaluated from the expression

$$C_{p1} = \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} \quad (2.7)$$

where all quantities on the right are now known.

It then follows from standard perfect gas relations that the gas constant  $R_1$  and the molecular weight  $W_1$  of the "equivalent perfect gas" are, respectively,

$$R_1 = \left( \frac{\gamma_1 - 1}{\gamma_1} \right) C_{p1} \quad (2.8)$$

and

$$W_1 = \frac{\bar{R}}{R_1} \quad (2.9)$$

where  $\bar{R}$  = universal gas constant

$$= 8317 \text{ joules/kg-mole } ^\circ\text{K}$$

Eqs. (2.6) through (2.9) complete the definition of the "equivalent perfect gas" which adequately simulates the essential thermodynamic properties of the primary vapor/liquid mixture.

It should also be explained that the secondary fluid at station 2 is predominantly a noncondensing gas of low molecular weight primary vapor.

It can be shown that the mass ratio  $\omega$  of condensible primary vapor to dry secondary gas at station 2 is given by the expression

$$\omega = \frac{W_V}{W_G} \frac{P_V(T_6)}{[P_2 - P_V(T_6)]} \quad (2.10)$$

where

$W_v$  = true molecular weight of condensible vapor (not to be confused with the "equivalent molecular weight  $W_1$ " considered earlier)

$W_G$  = molecular weight of noncondensing gas

$P_v(T_6)$  = vapor pressure of condensible primary fluid at known temperature  $T_6$

$P_2$  = known static pressure at station 2

The present analysis is restricted to conditions under which  $\omega$  is very small compared with unity and may be neglected. Thus the fluid at station 2 may be treated as dry gas. This greatly simplifies the analysis. Eq. (2.10) is useful for verifying that this assumption is indeed satisfied in any particular instance.

It should be added that if it later becomes of interest to make calculations for circumstances under which parameter  $\omega$  is not negligible, the present analysis can in fact be generalized to include this effect. For the present, however, we prefer to deal only with the simpler situation in which  $\omega$  may be neglected.

### 3. Ejector

The ejector receives a primary stream of high molecular weight gas at station 1, a secondary stream of low molecular weight gas at station 2 and discharges the resulting mixture at station 3. Static conditions at these three stations are designated by subscripts 1, 2, 3. Stagnation conditions at the corresponding stations are designated by subscripts s1, s2 and s3.

The following quantities are arbitrarily specified or known at stations 1 and/or 2, namely,

Molecular weights:  $W_1$  and  $W_2$

Ratios of specific heats:  $\gamma_1$  and  $\gamma_2$

Stagnation pressure of primary jet:  $P_{s1}$

Stagnation temperature of primary jet:  $T_{s1}$

Static pressures:  $P_1 = P_2 (= P_6)$

Static temperature of primary jet:  $T_1$  (= saturation temperature of primary fluid at pressure  $P_1$ )

Static temperature of secondary jet:  $T_2 (= T_6)$

Mass flow ratio:  $\dot{m}_1/\dot{m}_3 = \dot{m}_1/(\dot{m}_1 + \dot{m}_2) = x$

Velocity ratio:  $V_1/V_2 = y$

We treat the fluids at stations 1, 2, 3, 4 and 5 as perfect gases with constant specific heats.

The velocity ratio of the ejector may be developed as follows:

$$\frac{V_1}{V_2} = y = \frac{a_1}{a_2} \frac{M_1}{M_2} = \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \sqrt{\frac{M_1}{T_{s1}/T_1}} \cdot \frac{1}{M_2} \quad (3.1)$$

We can now use this result and other standard perfect gas relations to complete the calculation of key thermodynamic properties at stations 1 and 2. It is convenient to arrange these calculations sequentially as follows:

$$M_1 = \sqrt{\frac{2}{\gamma_1 - 1} \left( \frac{T_{s1}}{T_1} - 1 \right)} \quad (3.2)$$

$$M_2 = \left( \frac{V_2}{V_1} \right) \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \frac{M_1}{\sqrt{T_{s1}/T_1}} \quad (3.3)$$

$$\frac{T_{s2}}{T_2} = 1 + \frac{\gamma_2 - 1}{2} M_2^2 \quad (3.4)$$

$$T_{s2} = T_2 (T_{s2}/T_2) \quad (3.5)$$

$$\left( \frac{P_{s2}}{P_2} \right) = \left( \frac{T_{s2}}{T_2} \right)^{\frac{\gamma_2}{\gamma_2 - 1}} \quad (3.6)$$

$$P_{s2} = P_2 \left( \frac{P_{s2}}{P_2} \right) \quad (3.7)$$

Before we can determine the actual conditions at station 3, the outlet of the ejector, it is first necessary to analyze two other hypothetical cases as indicated schematically in Fig. 3.1. In all three of these cases the flow is taken as steady and adiabatic.

The first case, that shown in Fig. 3.1(a), involves a hypothetical device which receives two gas streams at stations 1 and 2 and discharges the resulting mixture at station x. Inlet conditions at 1 and 2 are identical to those of the actual ejector.

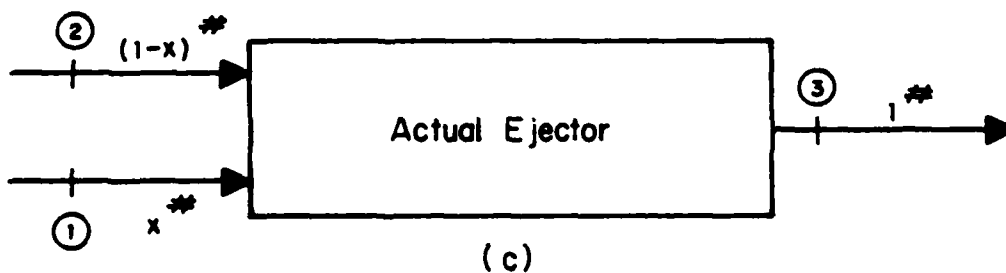
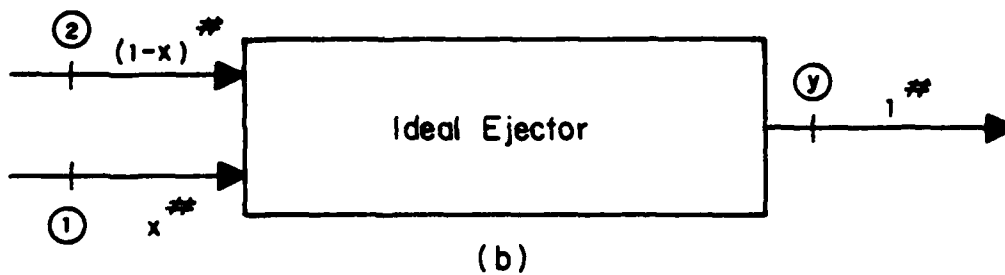
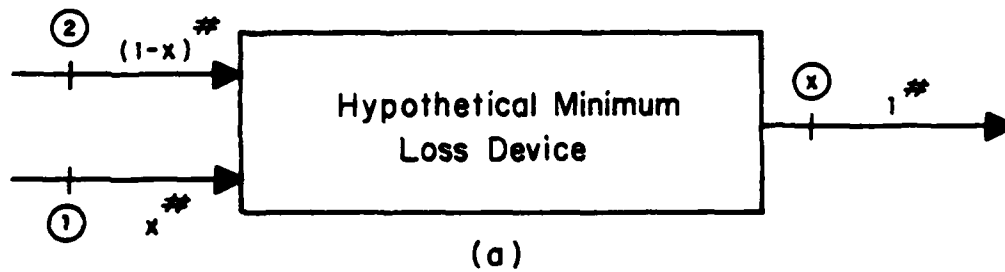


Fig. 3.1 COMPARISON OF ACTUAL EJECTOR WITH IDEAL EJECTOR AND WITH HYPOTHETICAL MINIMUM LOSS DEVICE.

Recall that the adiabatic mixing of two different gases is an inherently irreversible process that always involves a corresponding entropy increase. We stipulate that the first case is such that the only irreversibility which occurs is that associated with this mixing. We term this mass mixing to distinguish it from another type of mixing considered below which we term momentum mixing.

The second case, that shown in Fig. 3.1(b), involves an ideal ejector which receives two gas streams at stations 1 and 2 and discharges the resulting mixture at station y. Again, the inlet conditions at 1 and 2 are identical to those of the actual ejector. This case satisfies, among other relations, the idealized one-dimensional momentum equation for frictionless, constant area flow. It can be shown that in this case there is an overall entropy increase which includes not only the previously mentioned effect of mass mixing but also a further increase associated with momentum mixing.

The third case, that shown in Fig. 3.1(c), represents the actual ejector itself which receives input streams at stations 1 and 2 and which discharges the resulting mixture at station 3.

All three of the above cases refer to constant area processes in the sense that

$$A_x = A_y = A_3 = (A_1 + A_2) \quad (3.8)$$

Once the hypothetical operating conditions at stations x and y have been found, it then becomes possible to express the

corresponding actual conditions at station 3. This is accomplished through the use of an ejector effectiveness  $\eta_E$ . This parameter is defined more explicitly in the later analysis. Values of  $\eta_E$  must be estimated by reference to test data on ejectors (i.e., Ref. 8, 9, 10).



#### 4. Adiabatic Mixing of Two Streams

The process in the ejector is treated as the adiabatic mixing of two streams. The following relations can be shown to apply.

$$\bar{R} = 8315 \text{ Joule/kg-mole } ^\circ\text{K} \quad (4.1)$$

$$R_1 = \frac{\bar{R}}{W_1} \quad (4.2)$$

$$R_2 = \frac{\bar{R}}{W_2} \quad (4.3)$$

$$R_x = R_y = R_3 = x R_1 + (1 - x) R_2 \quad (4.4)$$

$$C_{p1} = \frac{\gamma_1 R_1}{(\gamma_1 - 1)} \quad (4.5)$$

$$C_{p2} = \frac{\gamma_2 R_2}{(\gamma_2 - 1)} \quad (4.6)$$

$$C_{px} = C_{py} = C_{p3} = x C_{p1} + (1 - x) C_{p2} \quad (4.7)$$

$$\gamma_x = \gamma_y = \gamma_3 = \frac{C_{p3}}{(C_{p3} - R_3)} \quad (4.8)$$

$$T_{sx} = T_{sy} = T_{s3} = \frac{1}{C_{p3}} [x C_{p1} T_{s1} + (1 - x) C_{p2} T_{s2}] \quad (4.9)$$

$$A_x = A_y = A_3 = (A_1 + A_2) \quad (4.10)$$

Notice that the quantities  $R$ ,  $C_p$ ,  $\gamma$ ,  $T_s$  and  $A$  all have the same values at stations  $x$ ,  $y$  and  $3$ .

This fact may be used to simplify many of the subsequent expressions. It is always permissible to substitute subscript  $3$  for subscripts  $x$  or  $y$  on any of these quantities.

## 5. Mass Flow

It is convenient temporarily to omit station subscripts 1, 2, 3, x, y and thereby develop certain needed mass flow relations in generalized form. Thus the mass flow across an arbitrary station may be written and developed as follows.

$$\begin{aligned}\dot{m} &= \rho AV = \left( \frac{P}{RT} \right) A (\sqrt{\gamma RT} M) = PAM \sqrt{\frac{\gamma}{RT}} \\ &= \frac{P_s A}{\sqrt{RT_s/\gamma}} \sqrt{\frac{T_s/T}{(P_s/P)}} M\end{aligned}\quad (5.1)$$

This may be rewritten as

$$\dot{m} = \frac{P_s A}{\sqrt{RT_s/\gamma}} f(M) \quad (5.2)$$

where the auxiliary function  $f(M)$  is defined as follows.

$$f(M) = \sqrt{\frac{T_s/T}{(P_s/P)}} M = M \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M^2 \right]^{-\frac{(\gamma + 1)}{2(\gamma - 1)}} \quad (5.3)$$

Eq. (5.2) can be applied specifically to stations 1, 2, x, y, 3 as follows

$$\dot{m}_1 = \frac{P_{s1} A_1}{\sqrt{R_1 T_{s1}/\gamma_1}} f(M_1) \quad (5.4)$$

$$\dot{m}_2 = \frac{P_{s2} A_2}{\sqrt{R_2 T_{s2}/\gamma_2}} f(M_2) \quad (5.5)$$

$$\dot{m}_x = \frac{P_{sx} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_x) \quad (5.6)$$

$$\dot{m}_y = \frac{P_{sy} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_y) \quad (5.7)$$

$$\dot{m}_3 = \frac{P_{s3} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_3) \quad (5.8)$$

also

$$\left(\frac{\dot{m}_1}{\dot{m}_3}\right) = x \quad (5.9)$$

$$\dot{m}_x = \dot{m}_y = \dot{m}_3 = \dot{m}_1 + \dot{m}_2 \quad (5.10)$$

From Eqs. (5.4), (5.5), (5.9) and (5.10) we readily find that

$$\left(\frac{A_1}{A_2}\right) = \frac{x}{(1-x)} \left(\frac{P_{s2}}{P_{s1}}\right) \sqrt{\frac{R_1 T_{s1} \gamma_2}{R_2 T_{s2} \gamma_1}} \frac{f(M_2)}{f(M_1)} \quad (5.11)$$

Then from Eq. (5.10) we infer also that

$$\left(\frac{A_1}{A_3}\right) = \frac{(A_1/A_2)}{(1 + A_1/A_2)} \quad (5.12)$$

Since all quantities on the right side of Eq. (5.11) are now known, Eqs. (5.11) and (5.12) fix the area ratios  $A_1/A_2$  and  $A_1/A_3$ .

From Eqs. (5.5), (5.6), (5.9), (5.10) and (3.8) we deduce further that

$$f(M_x) = \frac{(1 - A_1/A_3)}{(1 - x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{R_3 T_{s3} \gamma_2}{R_2 T_{s2} \gamma_3}} f(M_2) \quad (5.13)$$

All quantities on the right side of Eq. (5.13) except  $P_{sx}$  are now known. The method of determining  $P_{sx}$  is explained in a later section; refer to Eq. (5.15). Once  $P_{sx}$  has been found, Eq. (5.13) fixes  $f(M_x)$  whereupon Eq. (5.3) fixes  $M_x$  itself. The solution of Eq. (5.3) for  $M$  when  $f(M)$  is known involves an iterative procedure which is explained later in this section.

From Eqs. (5.6), (5.7), (5.8) and (5.10) we also find that

$$P_{sx} f(M_x) = P_{sy} f(M_y) = P_{s3} f(M_3) \quad (5.14)$$

Assuming  $P_{sx}$  and  $M_x$  known, this relation along with Eq. (5.3) fixes  $P_{sy}$  when  $M_y$  is specified; it also fixes  $f(M_3)$  and  $M_3$  when  $P_{s3}$  is specified. The methods of calculating  $M_y$  and  $P_{s3}$  are explained later; refer to Eqs. (7.8) and (8.14).

Fig. 5.1 is a rough sketch of the function  $f(M)$  as defined by Eq. (5.3). This function vanishes at  $M = 0$  and at  $M = \infty$ . The maximum value occurs at  $M = 1$  and equals

$$f_{\max} = \left[ \frac{2}{\gamma + 1} \right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} \quad (5.15)$$

It is clear from the figure that in the range  $0 < f < f_{\max}$ , Eq. (5.3) has two positive real roots, one subsonic and the other supersonic. Usually the supersonic root must be discarded because it is not consistent with a further constraint imposed by the second law of thermodynamics.

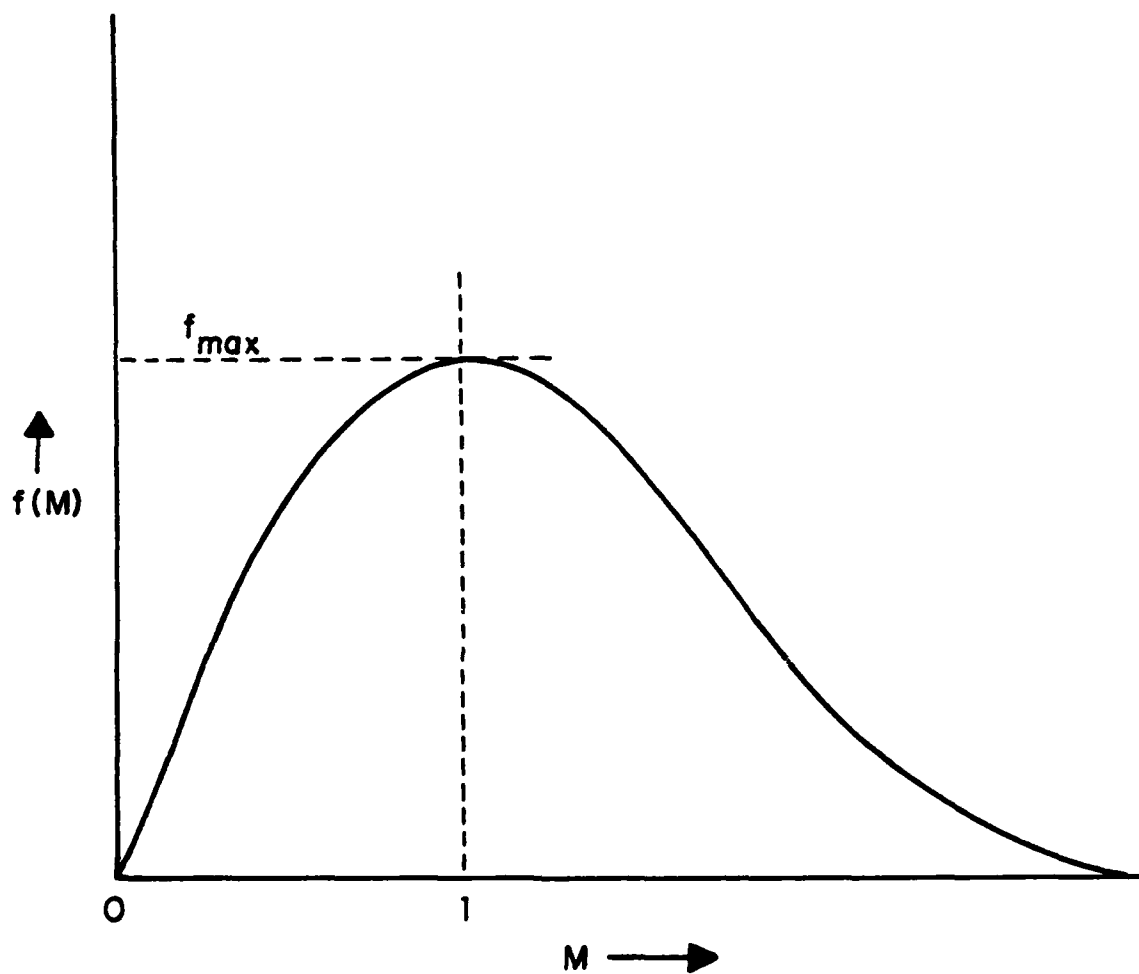


Fig. 5.1 APPROXIMATE SKETCH OF FUNCTION  $f(M)$

In order to solve Eq. (5.3) for  $M$  when values of  $f$  and  $\gamma$  are specified, we utilize an iterative procedure based on Newton's method. Let  $M_n$  denote the  $n$ th trial value of  $M$ . We then define a function  $F(M_n)$  and its derivative  $F'(M_n)$  as follows.

$$F(M_n) = f \left[ 1 + \frac{\gamma - 1}{2} M_n^2 \right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} - M_n \quad (5.16)$$

$$F'(M_n) = f \left( \frac{\gamma + 1}{2} \right) M_n \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_n^2 \right]^{\frac{(3 - \gamma)}{2(\gamma - 1)}} - 1 \quad (5.17)$$

The  $(n + 1)$ st trial value of  $M$  may now be taken as

$$M_{(n + 1)} = M_n - \frac{F(M_n)}{F'(M_n)} \quad (5.18)$$

The cycle of calculations defined by Eqs. (5.16), (5.17) and (5.18) is repeated until the result for  $M_n$  converges to a stable value at the desired level of accuracy.

Thus

$$M = \lim_{n \rightarrow \infty} M_n \quad (5.19)$$

It is readily apparent that the value of  $M$  so obtained satisfies Eq. (5.3) for the prescribed values of  $f$  and  $\gamma$ .

## 6. Entropy

Consider the case shown in Fig. 3.1(a). The specific entropy  $s_x$  of the discharged gas may be expressed in the form

$$s_x = x \left[ C_{p1} \ln \left( \frac{T_x}{T_o} \right) - R_1 \ln \left( \frac{P_{x1}}{P_o} \right) \right] + (1 - x) \left[ C_{p2} \ln \left( \frac{T_x}{T_o} \right) - R_2 \ln \left( \frac{P_{x2}}{P_o} \right) \right] \quad (6.1)$$

where  $P_{x1}$  and  $P_{x2}$  are the partial pressures of the two components of the gas mixture. Symbols  $P_o$  and  $T_o$  denote the pressure and temperature of the ambient atmosphere. The entropy of each component is assigned the value zero at this reference state  $P_o$ ,  $T_o$ .

Incidentally, the primary fluid, being condensible, might well exist only in the liquid state at the reference condition  $P_o$ ,  $T_o$ . Nevertheless, so long as it is in the gaseous state at station  $x$ , Eq. (6.1) may still be used.

The partial pressures of the two components are proportional to the respective mol fractions. Hence

$$\left( \frac{P_{x1}}{P_o} \right) = \frac{x R_1}{x R_1 + (1 - x) R_2} \left( \frac{P_x}{P_o} \right) = \frac{x R_1}{R_3} \left( \frac{P_x}{P_o} \right) \quad (6.2)$$

$$\left( \frac{P_{x2}}{P_o} \right) = \frac{(1 - x) R_2}{x R_1 + (1 - x) R_2} \left( \frac{P_x}{P_o} \right) = \frac{(1 - x) R_2}{R_3} \left( \frac{P_x}{P_o} \right) \quad (6.3)$$

Upon substituting Eqs. (6.2) and (6.3) into (6.1) and simplifying we may obtain the result in the form

$$s_x = s_m + C_{p3} \ln \left( \frac{T_x}{T_o} \right) - R_3 \ln \left( \frac{P_x}{P_o} \right) \quad (6.4)$$

where

$$s_m = R_3 \ln R_3 - x R_1 \ln (x R_1) - (1 - x) R_2 \ln [(1 - x) R_2] \quad (6.5)$$

It can be shown that this quantity  $s_m$  represents the entropy increase associated with the mass mixing of the two different gases.

Moreover, since the entropy of the stagnation state  $s_x$  is by definition identical to that of the corresponding static state  $x$ , we may replace  $T_x$  and  $P_x$  in Eq. (6.4) by  $T_{sx}$  and  $P_{sx}$ , respectively. Thus we obtain

$$s_x = s_m + C_{p3} \ln \left( \frac{T_{sx}}{T_o} \right) - R_3 \ln \left( \frac{P_{sx}}{P_o} \right) \quad (6.6)$$

In previous studies of this series, we have considered only the special case in which the same fluid is employed for both the primary and secondary streams. In that case we have

$$R_1 = R_2 = R_3 = R \quad (6.7)$$

$$C_{p1} = C_{p2} = C_{p3} = C_p \quad (6.8)$$

$$\gamma_1 = \gamma_2 = \gamma_3 = \gamma \quad (6.9)$$

Moreover, in this special case there is no mass mixing, so that we must set



$$s_m = 0 \quad (6.10)$$

Consequently, Eq. (6.6) now simplifies to

$$s_x = 0 + C_p \ln \left( \frac{T_{sx}}{T_o} \right) - R \ln \left( \frac{P_{sx}}{P_o} \right) \quad (6.11)$$

Next reverting to the case of two different gases, let  $\Delta t$  denote a time interval during which unit mass crosses station  $x$ . During this same interval  $x$  units of mass cross station 1 and  $(1 - x)$  units of mass cross station 2. The entropy which leaves across station  $x$  must equal the entropy which enters across stations 1 and 2 plus the entropy increase caused by the mass mixing. Thus

$$s_x = s_m + x s_1 + (1 - x) s_2 \quad (6.12)$$

where

$$s_1 = C_{p1} \ln \left( \frac{T_{s1}}{T_o} \right) - R_1 \ln \left( \frac{P_{s1}}{P_o} \right) \quad (6.13)$$

$$s_2 = C_{p2} \ln \left( \frac{T_{s2}}{T_o} \right) - R_2 \ln \left( \frac{P_{s2}}{P_o} \right) \quad (6.14)$$

Upon eliminating  $s_x$  between Eqs. (6.6) and (6.12), we find that  $s_m$  also cancels from the result. It is then a simple matter to solve for the exit stagnation pressure in the form

$$\ln \left( \frac{P_{sx}}{P_o} \right) = \frac{1}{R_3} \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - x s_1 - (1 - x) s_2 \right\} \quad (6.15)$$

Thus the sequence defined by Eqs. (6.13), (6.14) and (6.15) now fixes  $P_{sx}$ . Next referring back to Eq. (5.13), we can calculate the value of  $f(M_x)$ ; finally, from Eqs. (5.16), (5.17), (5.18) we can calculate  $M_x$  itself. Once  $P_{sx}$ ,  $T_{sx} = T_{s3}$  and  $M_x$  are known, it is a simple matter to calculate the corresponding conditions  $P_x$ ,  $T_x$  from the usual perfect gas relations.

## 7. Momentum

Consider the idealized one dimensional constant area ejector shown in Fig. 3.1(b). The momentum equation for this system may be written

$$(P_1 + \rho_1 V_1^2)A_1 + (P_2 + \rho_2 V_2^2)A_2 = (P_Y + \rho_Y V_Y^2)A_Y \quad (7.1)$$

The mass conservation relation for this system can be expressed in the form

$$\frac{\rho_1 A_1 V_1}{x} = \frac{\rho_2 A_2 V_2}{(1-x)} = \rho_Y A_Y V_Y \quad (7.2)$$

Let us divide the three terms of Eq. (7.1) by the corresponding three terms of Eq. (7.2). Notice that the areas cancel out of the result. In this way we obtain

$$x \frac{(P_1 + \rho_1 V_1^2)}{\rho_1 V_1} + (1-x) \frac{(P_2 + \rho_2 V_2^2)}{\rho_2 V_2} = \frac{(P_Y + \rho_Y V_Y^2)}{\rho_Y V_Y} \quad (7.3)$$

Temporarily dropping the station subscript, we next develop the typical term of Eq. (7.3) in generalized format as follows.

$$\begin{aligned} \frac{(P + \rho V^2)}{\rho V} &= \frac{\left[ P + \frac{P}{RT} (\gamma RT) M^2 \right]}{\frac{P}{RT} \sqrt{\gamma RT} M} = \sqrt{\frac{RT_s}{\gamma}} \frac{(1 + \gamma M^2)}{M \sqrt{T_s/T}} \\ &= \sqrt{\frac{RT_s}{\gamma}} \frac{(1 + \gamma M^2)}{M \sqrt{1 + \left(\frac{\gamma-1}{2}\right) M^2}} \end{aligned} \quad (7.4)$$

Observe that the pressure  $P$  cancels from the result.

It is convenient to define the auxiliary function

$$g(M) = \frac{M}{(1 + \gamma M^2)} \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M^2} \quad (7.5)$$

With this notation Eq. (7.3) may be rewritten in the form

$$\frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R_2 T_{s2}}{\gamma_2}} = \frac{1}{g(M_Y)} \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \quad (7.6)$$

Rearranging gives

$$g(M_Y) = \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \left\{ \frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R_2 T_{s2}}{\gamma_2}} \right\}^{-1} \quad (7.7)$$

Since all quantities on the right are known, Eq. (7.7) fixes  $g(M_Y)$ . Then  $M_Y$  itself can be found by inverting Eq. (7.5). Fortunately, an explicit solution is possible in this case as Eq. (7.5) can be reduced to a quadratic in  $M^2$ . The result is

$$M^2 = \frac{(1 - 2\gamma g^2) \pm \sqrt{1 - 2(\gamma + 1)g^2}}{1 - \gamma(1 - 2\gamma g^2)} \quad (7.8)$$

The general character of the function  $g(M)$  is sketched in Fig. 7.1. Notice that  $g(M)$  vanishes at  $M = 0$ , reaches its peak value  $g_{\max}$  at  $M = 1$ , and decreases toward the limit  $g_{\infty}$  as  $M \rightarrow \infty$ . It can be shown that

$$g_{\max} = \frac{1}{\sqrt{2(\gamma + 1)}} \quad (7.9)$$

$$g_{\infty} = \frac{1}{\gamma} \sqrt{\frac{(\gamma - 1)}{2}} \quad (7.10)$$

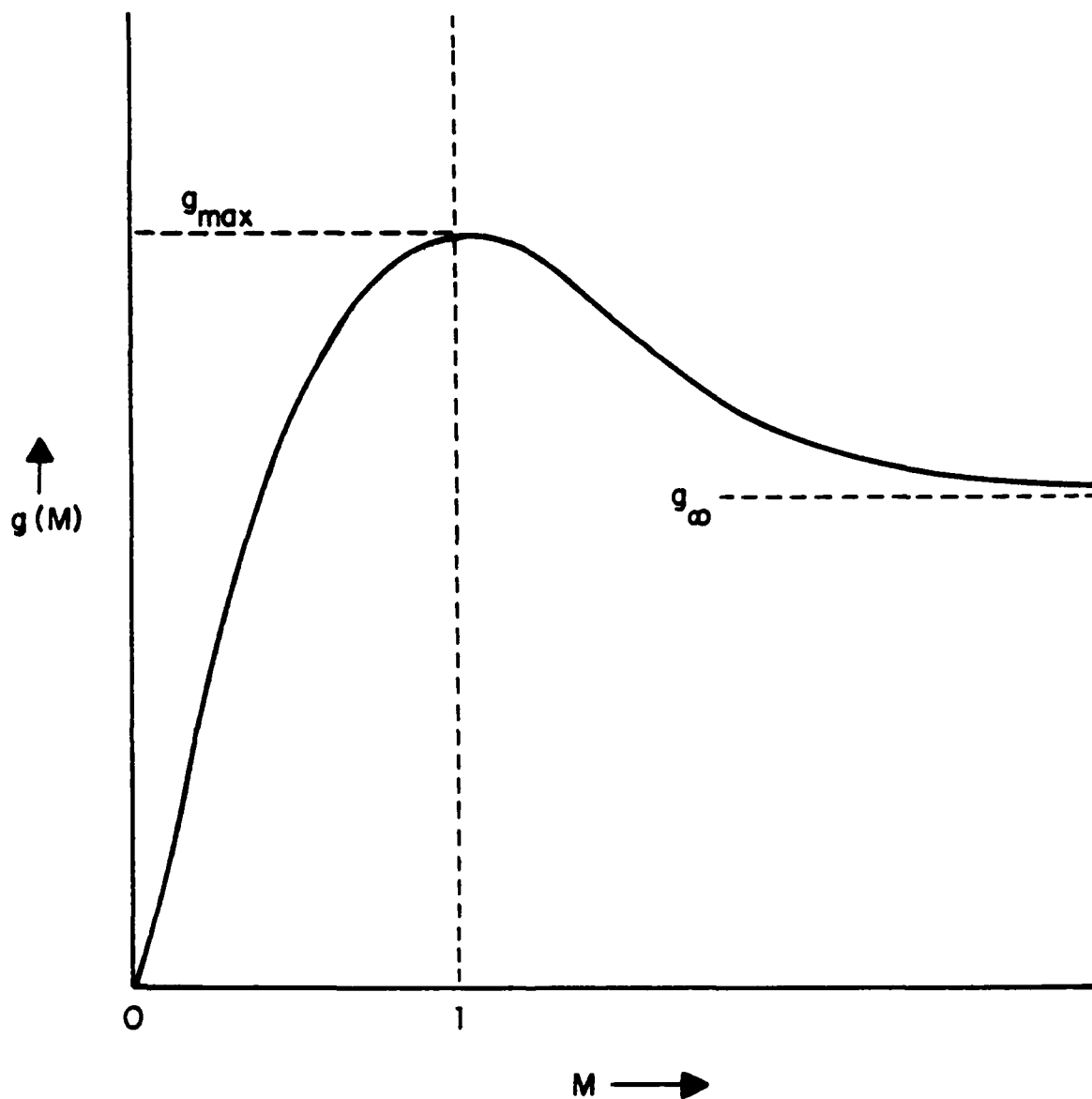


Fig. 7.1 APPROXIMATE SKETCH OF FUNCTION  $g(M)$

Fig. 7.1 discloses the existence of a supersonic root over the range  $g_{\infty} < g < g_{\max}$ . This root should not be accepted until a check calculation is made to determine whether the result is consistent with the second law of thermodynamics. The subsonic root is found by retaining only the negative sign before the radical in Eq. (7.8).

Once  $M_y$  has been found from Eq. (7.8), the corresponding value of  $P_{sy}$  may be found from Eqs. (5.3) and (5.14). This suffices to fix all properties at states  $y$  and  $sy$ .

## 8. Availability

The entropies  $s_1$  and  $s_2$  of the two input streams were defined earlier in Eqs. (6.13) and (6.14). The corresponding steady flow availability functions with respect to an ambient atmosphere at pressure  $P_0$  and temperature  $T_0$  may be written

$$\psi_1 = C_{p1} (T_{s1} - T_0) - T_0 s_1 \quad (8.1)$$

$$\psi_2 = C_{p2} (T_{s2} - T_0) - T_0 s_1 \quad (8.2)$$

Consequently the total available energy entering the system becomes

$$\psi_m = x \psi_1 + (1 - x) \psi_2 \quad (8.3)$$

The respective entropies of the streams leaving at stations  $x$ ,  $y$  and  $3$  are

$$s_x = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_0} \right) - R_3 \ln \left( \frac{P_{sx}}{P_0} \right) \quad (8.4)$$

$$s_y = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_0} \right) - R_3 \ln \left( \frac{P_{sy}}{P_0} \right) \quad (8.5)$$

$$s_3 = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_0} \right) - R_3 \ln \left( \frac{P_{s3}}{P_0} \right) \quad (8.6)$$

where  $s_m$  has been previously defined in Eq. (6.5).

The corresponding availabilities of the streams leaving at stations  $x$ ,  $y$  and  $3$  may therefore be written

$$\psi_x = C_{p3} (T_{s3} - T_o) - T_o s_x \quad (8.7)$$

$$\psi_y = C_{p3} (T_{s3} - T_o) - T_o s_y \quad (8.8)$$

$$\psi_3 = C_{p3} (T_{s3} - T_o) - T_o s_3 \quad (8.9)$$

Now consider the losses of availability listed below. By substituting the above expressions for the quantities on the left and simplifying we readily obtain the expressions shown on the right. Thus

$$(\psi_m - \psi_x) = T_o s_m \quad (8.10)$$

$$(\psi_x - \psi_y) = R_3 T_o \ln \left( \frac{P_{sx}}{P_{sy}} \right) \quad (8.11)$$

$$(\psi_x - \psi_3) = R_3 T_o \ln \left( \frac{P_{sx}}{P_{s3}} \right) \quad (8.12)$$

These results are very significant. Eq. (8.10) defines the loss of available energy caused by the mass mixing of the two different gases. Eq. (8.11) defines the further loss of available energy caused by the momentum mixing in an idealized ejector. Eq. (8.12) defines the augmented loss of available energy caused by the momentum mixing in the real ejector.

It is useful to postulate an empirical relation between these last two losses which is shown below on the left. The expression on the right then follows from Eqs. (8.11) and (8.12). Thus

$$\frac{(\psi_x - \psi_y)}{(\psi_x - \psi_3)} = \eta_E = \frac{\ln (P_{sy}/P_{sx})}{\ln (P_{s3}/P_{sx})} \quad (8.13)$$



We term  $\eta_E$  the ejector effectiveness. Its value, which is always less than unity, must be estimated from appropriate test data on ejectors.

If we treat  $\eta_E$  as known, Eq. (8.13) fixes  $P_{s3}$ . The solution is simply

$$P_{s3} = P_{sx} \left( \frac{P_{sy}}{P_{sx}} \right)^{\frac{1}{\eta_E}} \quad (8.14)$$

We can next find  $f(M_3)$  from Eq. (5.14) and  $M_3$  from Eqs. (5.16), (5.17) and (5.18). This suffices to fix all properties at states 3 and s3 thereby completing the analysis of the ejector.

## 9. Diffuser

It is convenient to analyze the diffuser in terms of the concept of availability in steady flow. However, we wish to deal with the characteristics of the diffuser itself and these cannot depend on the arbitrary values  $P_O$ ,  $T_O$  which happen to characterize the condition of the ambient atmosphere. We can achieve our objective by defining the availability with respect to a reference state  $P_3$ ,  $T_3$  which characterizes the diffuser rather than with respect to the ambient state  $P_O$ ,  $T_O$ .

Under these circumstances we may write the appropriate availabilities at diffuser inlet and outlet as follows

$$\psi_{s3} = C_{p3} (T_{s3} - T_3) - T_3 \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s3}}{P_3} \right) \right\} \quad (9.1)$$

$$\psi_{s4} = C_{p3} (T_{s3} - T_3) - T_3 \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s4}}{P_3} \right) \right\} \quad (9.2)$$

These equations make use of the fact that  $T_{s4} = T_{s3}$ . Also, states  $s3$  and  $3$  are at the same entropy so that

$$(s_{s3} - s_3) = 0 = C_p \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s3}}{P_3} \right) \quad (9.3)$$

From Eqs. (9.1) and (9.3) we conclude that

$$\psi_{s3} = C_{p3} (T_{s3} - T_3) = \left( \frac{V_3^2}{2} \right) \quad (9.4)$$

This result shows that the available energy at the diffuser inlet is simply the inlet kinetic energy itself.

The loss of availability through the diffuser may be found by subtracting Eq. (9.2) from (9.1). The result is

$$(\psi_{s4} - \psi_{s3}) = \Delta\psi = + R_3 T_3 \ln \left( \frac{P_{s3}}{P_{s4}} \right) \quad (9.5)$$

We now define the diffuser effectiveness as

$$\left( \frac{\psi_{s4}}{\psi_{s3}} \right) = \eta_D \quad (9.6)$$

so that

$$\left( \frac{\psi_{s3} - \psi_{s4}}{\psi_{s3}} \right) = \frac{\Delta\psi}{\psi_{s3}} = (1 - \eta_D) = \frac{R_3 T_3 \ln \left( \frac{P_{s3}}{P_{s4}} \right)}{C_{p3} (T_{s3} - T_3)} \quad (9.7)$$

This reduces readily to

$$(1 - \eta_D) = \frac{\ln \left( \frac{P_{s3}}{P_{s4}} \right)}{\left( \frac{\gamma_3 M_3^2}{2} \right)} \quad (9.8)$$

Solving for the pressure ratio gives

$$\frac{P_{s4}}{P_{s3}} = e^{- (1 - \eta_D) \frac{\gamma_3 M_3^2}{2}} \quad (9.9)$$

This is the result required. It fixes  $P_{s4}$  when  $P_{s3}$ ,  $M_3$  and  $\eta_D$  are specified.

Assuming the area ratio  $A_3/A_4$  to be arbitrarily specified, we can find  $M_4$  from the continuity relation. Referring to Eq. (5.8) we may write

$$\dot{m}_3 = \frac{P_{s3} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_3) = \frac{P_{s4} A_4}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_4) = \dot{m}_4 \quad (9.10)$$

Consequently

$$f(M_4) = \left( \frac{P_{s3}}{P_{s4}} \right) \left( \frac{A_3}{A_4} \right) f(M_3) \quad (9.11)$$

Eq. (9.11) fixes  $f(M_4)$ . Then  $M_4$  follows in the usual way from Eqs. (5.16), (5.17) and (5.18). The result suffices to fix all properties at states 4 and s4. Thus

$$\left( \frac{T_{s3}}{T_4} \right) = \left( \frac{T_{s4}}{T_4} \right) = 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \quad (9.12)$$

$$T_4 = T_{s3} / (T_{s3}/T_4) \quad (9.13)$$

$$\left( \frac{P_{s4}}{P_4} \right) = \left( \frac{T_{s3}}{T_4} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (9.14)$$

$$P_4 = P_{s4} / (P_{s4}/P_4) \quad (9.15)$$

The configurations studied in earlier work did not incorporate a diffuser. For the purpose of comparing the results of the present analysis with corresponding earlier work, it is desirable to be able to eliminate the effects of the diffuser in certain cases. This can be accomplished in the present analysis simply by setting

$$\eta_D = 1 \quad (9.16)$$

and

$$\left( \frac{A_3}{A_4} \right) = 1 \quad (9.17)$$

It is easy to confirm that under these circumstances

$$T_{s4} = T_{s3} \quad (9.18)$$

$$P_{s4} = P_{s3} \quad (9.19)$$

$$M_4 = M_3 \quad (9.20)$$

$$A_4 = A_3 \quad \text{etc.} \quad (9.21)$$

and all diffuser effects disappear.

## 10. Electrical Power Section

The power conversion process in an EHD duct is usually treated on the basis either of constant area or of constant static state. Inasmuch as electrical power output per unit mass is small, the numerical differences between the results computed by these two methods is negligible. For definiteness in this analysis, however, we assume constant area.

The electrical power that can be obtained from an EHD duct of constant area, negligible change of density and optimum length can be estimated from the one dimensional version of Poisson's equation which governs the electrical field. The solution is well known and will not be derived here; a detailed derivation may be found in Ref. (3). The essential result may be written in the form

$$P_e = \frac{1}{2} \epsilon E_b^2 A_4 V_4 \quad (10.1)$$

where

$P_e$  = gross electric power output, watts

$\epsilon$  = permittivity of medium

$= 8.854 \times 10^{-12}$  farad/m (for any gas)

$E_b$  = dielectric strength of medium, volts/m

$A_4$  = area of duct (constant)

$V_4$  = velocity

According to the test data analyzed in Ref. 4, the dielectric strength is well approximated by the expression

$$E_b = C_{B3} R_3 \rho_4 \quad (10.2)$$

where  $C_{B3}$  is a characteristic constant of the medium. The data show that Eq. (10.2) applies to air or steam up to about 10 atmospheres pressure with

$$C_{B3} \doteq 9.49 \times 10^3 \text{ m}^2 \text{ }^\circ\text{K/cmb} \quad (10.3)$$

In this study we assume that Eq. (10.2) can be extrapolated up to about 100 atmospheres pressure. We also assume that an expression of the same form applies to other media besides air or steam but that each medium has its own characteristic value of the breakdown constant  $C_B$ .

In the two-fluid system, the separate values of the breakdown constants  $C_{B1}$  and  $C_{B2}$  are usually known, but the breakdown constant  $C_{B3}$  of the resulting mixture is seldom known. In the absence of adequate test data bearing on this point, we tentatively assume that  $C_{B3}$  can be estimated from the hypothetical relation

$$C_{B3} = \frac{1}{R_3} [x R_1 C_{B1} + (1 - x) R_2 C_{B2}] \quad (10.4)$$

Upon substituting Eq. (10.2) into (10.1) and dividing through by the mass flow rate, we find the electrical work output per unit mass of fluid in the form:

$$\frac{P_e}{\dot{m}_4} = \left( \frac{\epsilon C_{B3}^2 R_3^2}{2} \right) \rho_4 = C_{p3} (T_{s4} - T_{s5}) \quad (10.5)$$

By use of standard perfect gas relations, Eq. (10.5) can be changed to a more useful form. In this connection set

$$(T_{s4} - T_{s5}) = \Delta T_s \quad (10.6)$$

and

$$\beta = \frac{\epsilon C_{B3}^2 P_o}{T_o^2} \quad (10.7)$$

where

$$P_o = \text{ambient pressure, N/m}^2$$

$$T_o = \text{ambient temperature, } ^\circ\text{K}$$

Notice that  $\beta$  represents a dimensionless version of the electrical breakdown constant.

Eq. 10.5 now yields the important result

$$\frac{\Delta T_s}{T_{s4}} = \beta \left( \frac{\gamma_3 - 1}{2\gamma_3} \right) \left( \frac{P_{s4}}{P_o} \left( \frac{T_o}{T_{s4}} \right) \right)^2 \left( 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \right)^{-\frac{1}{\gamma_3 - 1}} \quad (10.8)$$

The corresponding gross electrical work output per unit mass of fluid is then simply

$$W_e^* = C_{p3} \Delta T_s \quad (10.9)$$

Unfortunately, the known value of  $\epsilon$  and the typical experimentally measured values of  $C_B$  are very small. Consequently, the dimensionless breakdown constant  $\beta$  is extremely small compared to unity. Hence the drop in stagnation temperature  $\Delta T_s$  and the gross electrical work per unit mass  $W_e^*$  are also typically very small. This is a very basic and serious limitation on the performance that can be obtained from an EHD generator.

Treating the process 4+5 as isentropic, we may write



$$P_{s5} = P_{s4} \left( \frac{T_{s5}}{T_{s4}} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (10.10)$$

By analogy with Eq. (5.8) we write the continuity relation as

$$\dot{m}_4 = \frac{P_{s4} A_4}{\sqrt{R_3 T_{s4}/\gamma_3}} f(M_4) = \frac{P_{s5} A_5}{\sqrt{R_3 T_{s5}/\gamma_3}} f(M_5) = \dot{m}_5 \quad (10.11)$$

It then follows that

$$f(M_5) = \sqrt{\frac{T_{s5}}{T_{s4}}} \left( \frac{P_{s4}}{P_{s5}} \right) f(M_4) \quad (10.12)$$

This fixes  $f(M_5)$  whereupon  $M_5$  may be found in the usual way from Eqs. (5.16), (5.17) and (5.18). However, convergence should now be very rapid since we may take as a first approximation

$$M_5 \approx M_4 \quad (10.13)$$

This solution now suffices to fix all properties at states 4 and s4.

## 11. Condenser/Separator

The primary fluid which leaves the condenser/separator at station 6 is assumed to be compressed liquid at known static pressure  $P_6 = P_2 = P_1$  and at known static temperature  $T_6 = T_2$ . The kinetic energy of the liquid at station 6 is assumed to be negligible.

Temperature  $T_6$  must be equal to or greater than the ambient temperature  $T_0$  in order to satisfy the requirements of a heat balance on the condenser/separator. No attempt is made in this analysis to formulate this actual heat balance; instead  $T_6$  is simply treated as a given or known quantity. However, it may be of interest later to investigate the effects of varying  $T_6$  on the overall performance of the EHD system.

The secondary fluid which leaves the condenser/separator at station 2 consists primarily of a low molecular weight noncondensing gas. It also contains some secondary vapor, but the mass fraction of this vapor is treated as negligible in this analysis. Thus the secondary fluid at station 2 is treated as dry gas.

The thermodynamic process in the condenser/separator can be most conveniently analyzed and modelled by employing the momentum theorem. We treat the condenser/separator essentially as a one dimensional, constant area device in steady flow. By equating the net applied pressure and drag forces to the change of momentum flux in the direction of flow, we obtain the following simple relation, namely,

$$(p_5 - p_z)A_5 - D = (1 - x)\dot{m}_3 V_z - \dot{m}_3 V_5 \quad (11.1)$$

The effective drag force  $D$  in this equation represents two irreversible momentum loss effects. One of these is the loss of momentum produced when the condensible vapor in the incoming flow is condensed and brought to a halt on the stationary surfaces of the condenser/separator. The other is the additional friction drag force; the latter is conveniently expressed in terms of an empirical friction coefficient  $c_f$ . Thus we may write

$$D = x \dot{m}_3 V_5 + c_f \left( \frac{1}{2} \rho_5 V_5^2 \right) A_5 \quad (11.2)$$

Next we substitute Eq. (11.2) into 11.1), bring all terms to one side of the equation, regroup terms, and divide through by the quantity  $p_5 A_5$ . We also make use of the following auxiliary relationships, namely,

$$\frac{p_5 V_5^2 A_5}{p_5 A_5} = \frac{\dot{m}_3 V_5}{p_5 A_5} = \gamma_3 M_5^2 \quad (11.3)$$

and

$$\frac{\dot{m}_3 V_z}{p_5 A_5} = \gamma_3 M_5 M_z \frac{\gamma_2 R_2 T_z}{\gamma_3 R_3 T_5} \quad (11.4)$$

In this way the following result is finally obtained -

$$E(x) = \left(1 - \frac{p_z}{p_5}\right) + \left(1 - x - \frac{c_f}{2}\right) \gamma_3 M_5^2 - (1 - x) M_5 M_z \sqrt{\frac{\gamma_2 R_2 T_z}{\gamma_3 R_3 T_5}} = 0 \quad (11.5)$$

The quantity  $E(x)$  serves here merely as a convenient abbreviation for the sum of terms shown. It may be said to represent an excess of momentum available over that required to satisfy the various losses that occur across the condenser/separator. In any case, for assigned values of all other parameters, we seek by successive trials a value of  $x$  such that  $E(x) = 0$ . Thus Eq. (11.5) fixes the mass fraction  $x$  in a manner which is consistent with the other specified input parameters.

Of course, if the various fixed input parameters are not suitably chosen, Eq. (11.5) cannot be satisfied for any trial value of  $x$  in the range  $0 < x < 1$ , and no solution is physically possible under these conditions.

## 12. Cycle Efficiency

Once a solution has been found such that  $E(x) = 0$ , it becomes a straightforward matter to calculate the corresponding overall cycle efficiency.

Because of the pressure drop through the boiler/superheater, the pressure  $P_7 = P_{s7}$  at boiler inlet is slightly higher than the stagnation pressure  $P_{s1}$  of the primary fluid at the ejector inlet. We assume that the ratio  $P_{s1}/P_{s7}$  is a specified constant. Hence with  $P_{s1}$  specified,  $P_{s7}$  is also known. Also the pressures  $P_6 = P_1$  are known.

Consider the ideal pump work  $w_p^*$  done per unit mass of primary fluid. For the present consider the hypothetical case of a reversible pump. Also note that density changes across the pump are negligible. Hence we may write

$$w_p^* = \frac{(P_{s7} - P_6)}{\rho_6} \quad (12.1)$$

The ideal gross electrical work output per unit mass of mixed fluid has earlier been established in Eq.(10.5). It is denoted by symbol  $w_e^*$ .

Let us now denote the net useful electrical work output per unit mass of primary fluid by symbol  $w_{net}$ . It may be related to  $w_e^*$  and  $w_p^*$  in the following way

$$w_{net} = \left( \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} \right) \text{ Joule/kg} \quad (12.2)$$

where  $\eta_x$ , the excitation efficiency, allows for the small electrical power expended to excite the system and where  $\eta_p$ ,

the pump efficiency, allows for irreversibilities in the pump.

The heat input in the boiler/superheater per unit mass of primary fluid may be written

$$q_{in} = (h_{sl} - h_6) - \frac{w_p^*}{\eta_p} \quad (12.3)$$

where  $h_6$  is the enthalpy of the primary liquid at conditions  $P_6$ ,  $T_6$  as listed in suitable tables of properties.

The overall thermal efficiency of the cycle can now be calculated from the simple formula

$$\eta_c = \frac{w_{net}}{q_{in}} \quad (12.4)$$

Both  $\eta_c$  and  $w_{net}$  are useful parameters which characterize the overall thermodynamic performance of the EHD system.

Eq. (12.2) shows clearly how the ejector serves as a kind of amplifier which increases the electrical work output per unit mass of primary fluid. This effect is shown by the presence of the parameter  $x$  in the denominator of the first term. Notice the beneficial effect of a low value of  $x$  on parameters  $w_{net}$  and  $\eta_c$ . Unfortunately, the value of  $x$  cannot be stipulated independently in advance; it is fixed by the other specified input parameters as explained in the previous section.

Notice that owing to the fact that  $w_e^*$  is normally very small, it is possible in some circumstances for  $w_{net}$

to take on values which are actually negative. When this happens it means that electrical power output is insufficient to drive the pump.

The relationships developed in this and the preceding sections make it possible to carry out systematic parametric studies of various one-fluid and two-fluid EHD cycles, with realistic allowances for the various losses that occur. Such studies can establish optimum design parameters and performance limits under various circumstances. These results in turn can finally permit informed conclusions to be drawn concerning the ultimate feasibility of this general type of EHD power generator.

### 13. Calculation Sequence

In this section we summarize the various equations previously derived in the approximate order in which they would be used in the calculation of system performance. Also listed are the initial input parameters whose values must be specified in order to start the calculation and various further input parameters whose values must be specified in order to proceed with various successive stages of the calculation.

Initial Input Data:  $P_1 = P_2 = P_6$  ,  $T_1$  ,  $P_{s1}$  ,  $T_{s1}$  ,  $(h_{s1} - h_1)$  ,  
 $\bar{R} = 8315 \text{ Joule/kg}^\circ\text{K}$

$$\gamma_1 = \left( 1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)} \right)^{-1} \quad (13.1)$$

$$C_{p1} = \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} \quad (13.2)$$

$$R_1 = \left( \frac{\gamma_1 - 1}{\gamma_1} \right) C_{p1} \quad (13.3)$$

$$W_1 = \frac{\bar{R}}{R_1} \quad (13.4)$$

Further Input Data:  $W_v$  ,  $W_G = W_2$  ,  $T_2 = T_6$  ,  $P_v(T_6)$

$$\omega = \frac{W_v}{W_G} \frac{P_v(T_6)}{[P_2 - P_v(T_6)]} \quad (13.5)$$

Verify that  $\omega < 1$



Further Input Data:  $\gamma_2, \left(\frac{V_1}{V_2}\right) = y$

$$M_1 = \sqrt{\frac{2}{(\gamma_1 - 1)} \left[ \left( \frac{T_{s1}}{T_1} \right) - 1 \right]} \quad (13.6)$$

$$M_2 = \left( \frac{V_2}{V_1} \right) \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \frac{M_1}{\sqrt{T_{s1}/T_1}} \quad (13.7)$$

Verify that  $M_2 < 1$  (13.8)

$$\left( \frac{T_{s2}}{T_2} \right) = 1 + \left( \frac{\gamma_2 - 1}{2} \right) M_2^2 \quad (13.9)$$

$$T_{s2} = T_2 (T_{s2}/T_2) \quad (13.10)$$

$$\left( \frac{P_{s2}}{P_2} \right) = \left( \frac{T_{s2}}{T_2} \right)^{\frac{\gamma_2}{(\gamma_2 - 1)}} \quad (13.11)$$

$$P_{s2} = P_2 \left( \frac{P_{s2}}{P_2} \right) \quad (13.12)$$

Further Input Data:  $x$  (trial value)

$$R_2 = \frac{\bar{R}}{W_2} \quad (13.13)$$

$$R_3 = x R_1 + (1 - x) R_2 \quad (13.14)$$

$$C_{p2} = \frac{\gamma_2 R_2}{(\gamma_2 - 1)} \quad (13.15)$$

$$C_{p3} = x C_{p1} + (1 - x) C_{p2} \quad (13.16)$$

$$\gamma_3 = \frac{C_{p3}}{(C_{p3} - R_3)} \quad (13.17)$$

$$T_{s3} = \frac{1}{C_{p3}} [x C_{p1} T_{s1} + (1 - x) C_{p2} T_{s2}] \quad (13.18)$$

$$f(M_1) = M_1 \left[ 1 + \frac{\gamma_1 - 1}{2} M_1^2 \right]^{-\frac{(\gamma_1 + 1)}{2(\gamma_1 - 1)}} \quad (13.19)$$

$$f(M_2) = M_2 \left[ 1 + \frac{\gamma_2 - 1}{2} M_2^2 \right]^{-\frac{(\gamma_2 + 1)}{2(\gamma_2 - 1)}} \quad (13.20)$$

$$\frac{A_1}{A_2} = \frac{x}{(1 - x)} \left( \frac{P_{s2}}{P_{s1}} \right) \sqrt{\frac{R_1 T_{s1} \gamma_2}{R_2 T_{s2} \gamma_1} \frac{f(M_2)}{f(M_1)}} \quad (13.21)$$

$$\frac{A_1}{A_3} = \frac{(A_1/A_2)}{(1 + A_1/A_2)} \quad (13.22)$$

Further Input Data:  $P_o$  ,  $T_o$

$$s_1 = C_{p1} \ln \left( \frac{T_{s1}}{T_o} \right) - R_1 \ln \left( \frac{P_{s1}}{P_o} \right) \quad (13.23)$$

$$s_2 = C_{p2} \ln \left( \frac{T_{s2}}{T_o} \right) - R_2 \ln \left( \frac{P_{s2}}{P_o} \right) \quad (13.24)$$

$$\ln \left( \frac{P_{sx}}{P_o} \right) = \frac{1}{R_3} \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - x s_1 - (1 - x) s_2 \right\} \quad (13.25)$$

$$P_{sx} = P_o e^{\ln (P_{sx}/P_o)} \quad (13.26)$$

$$f(M_x) = f \frac{(1 - A_1/A_3)}{(1 - x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{R_3 T_{s3} \gamma_2}{R_2 T_{s2} \gamma_3}} f(M_2) \quad (13.27)$$

$$f_{\max} = \left( \frac{2}{\gamma_3 + 1} \right)^{\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} \quad (13.28)$$

$$\text{Verify that } f(M_x) = f_{\max} \quad (13.29)$$

$$\text{For first approximation set } M_x = M_2 \quad (13.30)$$

Iterate using Eqs. (13.31), (13.32), (13.33).

$$F(M_n) = f \left( 1 + \frac{(\gamma_3 - 1)}{2} M_n^2 \right)^{\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} - M_n \quad (13.31)$$

$$F'(M_n) = f \frac{(\gamma_3 + 1)}{2} M_n \left( 1 + \frac{(\gamma_3 - 1)}{2} M_n^2 \right)^{\frac{(3 - \gamma_3)}{2(\gamma_3 - 1)}} \quad (13.32)$$

$$M_{(n+1)} = M_n - \frac{F(M_n)}{F'(M_n)} \quad (13.33)$$

Iteration converges to fix  $M_x$

$$g(M_1) = \frac{M_1}{(1 + \gamma_1 M_1^2)} \sqrt{1 + \left( \frac{\gamma_1 - 1}{2} \right) M_1^2} \quad (13.34)$$

$$g(M_2) = \frac{M_2}{(1 + \gamma_2 M_2^2)} \sqrt{1 + \left( \frac{\gamma_2 - 1}{2} \right) M_2^2} \quad (13.35)$$

$$g(M_Y) = g = \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \left( \frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R T_{s2}}{\gamma_2}} \right)^{-1} \quad (13.36)$$

$$M_Y^2 = \frac{(1 - 2\gamma_3 g^2) - \sqrt{1 - 2(\gamma_3 + 1)g^2}}{1 - \gamma_3(1 - 2\gamma_3 g^2)} \quad (13.37)$$

$$\text{Eq. (13.37) applies provided that } g \neq g_\infty = \frac{1}{\gamma_3} \sqrt{\frac{(\gamma_3 - 1)}{2}} \quad (13.38)$$

If  $g = g_\infty$  the solution reduces to

$$M_Y = \sqrt{\frac{(\gamma_3 - 1)}{2\gamma_3}} \quad (13.39)$$

Once  $M_Y$  is known from Eq. (13.37) or (13.39)

$$f(M_Y) = M_Y \left( 1 + \frac{(\gamma_3 - 1)}{2} M_Y^2 \right)^{-\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} \quad (13.40)$$

$$P_{sy} = P_{sx} \frac{f(M_x)}{f(M_Y)} \quad (13.41)$$

Further Input Data:  $\eta_E$

$$P_{s3} = P_{sx} \left( \frac{P_{sy}}{P_{sx}} \right)^{1/\eta_E} \quad (13.42)$$

$$f(M_3) = \left( \frac{P_{sx}}{P_{s3}} \right) f(M_x) \quad (13.43)$$

$$\text{Verify that } f(M_3) = f < f_{\text{Max}} \quad (13.44)$$

For a first approximation take  $M_3 = M_y$  (13.45)

Iterate using Eqs. (13.31); (13.32), (13.33).

Iteration converges to fix  $M_3$ .

$$\left(\frac{T_{s3}}{T_3}\right) = \left[1 + \frac{(\gamma_3 - 1)}{2} M_3^2\right] \quad (13.46)$$

$$T_3 = T_{s3} / (T_{s3} / T_3) \quad (13.47)$$

$$\left(\frac{P_{s3}}{P_3}\right) = \left(\frac{T_{s3}}{T_3}\right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (13.48)$$

$$P_3 = P_{s3} / (P_{s3} / P_3) \quad (13.49)$$

Further Input Data:  $\eta_D, \left(\frac{A_3}{A_4}\right)$

$$P_{s4} = P_{s3} e^{-\left(1 - \eta_D\right)\left(\frac{\gamma_3 M_3^2}{2}\right)} \quad (13.50)$$

$$f(M_4) = f = \left(\frac{P_{s3}}{P_{s4}}\right)\left(\frac{A_3}{A_4}\right) f(M_3) \quad (13.51)$$

Verify that  $f(M_4) = f < f_{\max}$  (13.52)

For a first approximation take  $M_4 = \left(\frac{A_3}{A_4}\right) M_3$  (13.53)

Iterate using Eqs. (13.31), (13.32), (13.33).

Iteration converges to fix  $M_4$ .

$$T_{s4} = T_{s3} \quad (13.54)$$

$$\frac{T_{s4}}{T_4} = 1 + \frac{(\gamma_3 - 1)}{2} M_4^2 \quad (13.55)$$

$$T_4 = T_{s4} / (T_{s4} / T_4) \quad (13.56)$$

$$\frac{P_{s4}}{P_4} = \frac{T_{s4}}{T_4}^{\frac{\gamma_3}{\gamma_3 - 1}} \quad (13.57)$$

$$P_4 = P_{s4} / (P_{s4} / P_4) \quad (13.58)$$

Further Input Data:  $\epsilon = 8.854 \times 10^{-12}$  farad/m,  $C_{B1}$ ,  $C_{B2}$ ,

( $C_B = 9.5 \times 10^3 \text{ m}^2 \text{ }^\circ\text{K/cmb}$  for air or steam)

$$C_{B3} = \frac{1}{R_3} [x R_1 C_{B1} + (1 - x) R_2 C_{B2}] \quad (13.59)$$

$$\beta = \frac{\epsilon C_{B3}^2 P_O}{T_O^2} \quad (13.60)$$

$$\frac{\Delta T_s}{T_{s4}} = \beta \left( \frac{\gamma_3 - 1}{2\gamma_3} \right) \left( \frac{P_{s4}}{P_O} \right) \left( \frac{T_O}{T_{s4}} \right)^2 \left( 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \right)^{-1/(\gamma_3 - 1)} \quad (13.61)$$

$$\frac{T_{s5}}{T_{s4}} = \left( 1 - \frac{\Delta T_s}{T_{s4}} \right) \quad (13.62)$$

$$P_{s5} = P_{s4} \left( \frac{T_{s5}}{T_{s4}} \right)^{\gamma_3/(\gamma_3 - 1)} \quad (13.63)$$

$$f(M_5) = f = \sqrt{\frac{T_{s5}}{T_{s4}}} \left( \frac{P_{s4}}{P_{s5}} \right) f(M_4) \quad (13.64)$$

$$\text{Verify that } f(M_5) = f < f_{\max} \quad (13.65)$$

$$\text{For a first approximation take } M_5 = M_4 \quad (13.66)$$

Iterate using Eqs. (13.31), (13.32), (13.33)

Iteration converges to fix  $M_5$ .

$$\left( \frac{T_{s5}}{T_5} \right) = 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_5^2 \quad (13.67)$$

$$T_5 = T_{s5} / (T_{s5} / T_5) \quad (13.68)$$

$$\left( \frac{P_{s5}}{P_5} \right) = \left( \frac{T_{s5}}{T_5} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (13.69)$$

$$P_5 = P_{s5} / (P_{s5} / P_5) \quad (13.70)$$

Further Input Data:  $c_f$

$$E(x) = \left( 1 - \frac{P_z}{P_5} \right) + \left( 1 - x - \frac{c_f}{2} \right) \gamma_3 M_5^2 \quad (13.71)$$

$$- (1 - x) \gamma_3 M_5 M_z \sqrt{\frac{\gamma_2}{\gamma_3} \cdot \frac{R_2}{R_3} \cdot \frac{T_z}{T_5}} = 0$$

Iterate from Eq. (13.14) until  $x$  converges.

Further Input Data:  $(P_{s1}/P_{s7})$  ,  $\rho_6$  ,  $(h_{s1} - h_6)$  ,  $\eta_x$  ,  $\eta_p$

$$P_{s7} = P_{s1}/(P_{s1}/P_{s7}) \quad (13.72)$$

$$w_e^* = C_{p3} \Delta T_s \quad (13.73)$$

$$w_p^* = \frac{(P_{s7} - P_6)}{\rho_6} \quad (13.74)$$

$$w_{net} = \left( \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} \right) \quad \underline{\underline{RESULT}} \quad (13.75)$$

$$q_{in} = (h_{s1} - h_6) - \frac{w_p^*}{\eta_p} \quad (13.76)$$

$$\eta_c = \frac{w_{net}}{q_{in}} \quad \underline{\underline{RESULT}} \quad (13.77)$$



#### 14. OPTIMUM VELOCITY RATIO

The calculation sequence summarized in the preceding section reveals that a complete cycle calculation requires us to specify numerical values for about thirty input parameters. One of these is the velocity ratio  $(V_1/V_2) = y$ . The calculation procedure outlined above then permits us to find the corresponding value of the mass flow ratio  $x$ , assuming that there exists a value of  $x$  which satisfies the governing thermodynamic relations for the specified values of the other input parameters.

If we now change the value of  $y$  but hold all other input parameters fixed, we can repeat the above procedure and find a corresponding new value of  $x$ , if such a value exists. It is evident, therefore, that  $x$  becomes some definite function of  $y$  as long as we remain in the domain where a real solution exists. Of course all other dependent variables of the cycle are also functions of  $y$ . In particular, the overall cycle efficiency  $\eta_c$  is some definite function of  $y$ . Moreover, for prescribed values of the other input parameters, there will be some definite value of  $y$ , let us designate it as the optimum value  $y_{opt}$ , which yields the greatest value of  $\eta_c$  that is possible under the specified input conditions. Our basic purpose therefore is to determine for any prescribed values of the other input parameters, the value of  $y_{opt}$  and the corresponding value of  $\eta_c)_{max}$ . Of course the values of  $x_{opt}$  and other dependent variables are also of interest.

Since a working computer program is now available to establish values of  $y_{opt}$ ,  $\eta_c)_{max}$ ,  $x_{opt}$  and so on for prescribed values of the other input parameters, we can proceed to study the effects of changes in these other parameters and to search systematically for such values of the most important input parameters as will yield the best overall performance of the system.

## 15. Conclusions and Recommendations

Based on the work reported here and in Ref. (6), the following conclusions are evident:

- 1) Ejector performance is substantially improved through the use of a high molecular weight primary fluid (i.e., above 100) and a low molecular weight secondary (i.e., below 30). The mixing penalty which ensues is offset by the substantial decrease in ejector losses.
- 2) The breakdown strength limitation is severe even at higher pressures. Since the breakdown strength of a mixture can often be smaller than that of a single fluid, allowance for this must be made when considering fluids to be used in the ejector. In Ref. (6) it is stated that in mercury/hydrogen an improvement by a factor of 5 in the breakdown strength is required for a competitive thermodynamic efficiency.
- 3) Consideration of the breakdown strength of mixtures together with other factors seem to indicate that the sequence of components shown in Fig. 1.1 might not be the best possible. In particular, serious thought should be given to an arrangement where the electrical section precedes the ejector. Here, mostly secondary fluid is present and the breakdown strength will be at its highest value.

The termination of the contract with DoE has left some questions about the two-fluid cycle unanswered. With the analysis presented in this report it should be possible to carry out further thermodynamic studies with various fluid combinations. Also, more fundamental knowledge on the breakdown strength of dielectric fluids will be necessary before the prospects for successful EHD power generation can be fully evaluated.

#### 16. Sample Output

A sample output is shown in the following pages. The case is for mercury/hydrogen expanding from 1500 psia to 700 psia. The value of the breakdown constant,  $C_B$ , is 8000. The component loss parameters have numerical values corresponding to CASE B in Ref. 6.

# FLUID PROPERTIES

```

1 PS1= 1500.0 PSIA          2 TS1= 1945.0 DEG R
3 P1= 700.0 PSIA           4 T1= 1730.0 DEG R
5 (HS1-H1)= 13.50 BTU/LB    6 (HS1-HA)= 130.70 BTU/LB
7 RHQ6= 0.7500E 03 LB/FT3  8 CR1= 0.8000E 04 M2 DEG K/CMR
9 NFLUID= 2                 10 MG= 0.2014E 01
11 GAMMA2= 1.4000           12 CR2= 0.8000E 04 M2 DEG K/CMR

```

## SYSTEM PARAMETERS

```

1 ETAE= 0.85      2 ETAD= 0.85      3 A3/A4=0.250      4 CF= 0.80
5 ETAX= 0.98      6 ETAP= 0.85      7 PS1/PS7= 1.00

```

## COMPUTATIONAL FORMAT

```

1 DELY= 0.10      2 YMIN= 1.00      3 YMAX= 5.00

```

```

WV= 0.2058E 03(EFF MOL WT VAPOR)  GAMMA1= 1.1816  Y2CR= 0.1064

```

## ENTER CONTROL CODE

>-4

Y	X	A1/A3	OVERALL EFF	RELATIVE EFF	M3	M5
5.00	0.4375	0.0015	0.0228	0.2285	0.0285	0.0071
4.90	0.4531	0.0017	0.0213	0.2135	0.0295	0.0074
4.80	0.4375	0.0016	0.0228	0.2285	0.0294	0.0074
4.70	0.4375	0.0016	0.0228	0.2285	0.0303	0.0074
4.60	0.4277	0.0016	0.0238	0.2384	0.0307	0.0077
4.50	0.4063	0.0015	0.0261	0.2620	0.0308	0.0077
4.40	0.4219	0.0016	0.0244	0.2446	0.0319	0.0080
4.30	0.4063	0.0016	0.0261	0.2620	0.0322	0.0080
4.20	0.4063	0.0016	0.0261	0.2620	0.0329	0.0082
4.10	0.3906	0.0015	0.0280	0.2807	0.0333	0.0083
4.00	0.3906	0.0016	0.0280	0.2808	0.0341	0.0085
3.90	0.3750	0.0015	0.0300	0.3011	0.0346	0.0086
3.80	0.3594	0.0014	0.0322	0.3232	0.0350	0.0088
3.70	0.3672	0.0015	0.0311	0.3119	0.0362	0.0091
3.60	0.3594	0.0015	0.0322	0.3232	0.0370	0.0092
3.50	0.3438	0.0015	0.0346	0.3473	0.0376	0.0094
3.40	0.3477	0.0015	0.0340	0.3411	0.0388	0.0097
3.30	0.3398	0.0015	0.0352	0.3537	0.0397	0.0099
3.20	0.3281	0.0015	0.0372	0.3737	0.0406	0.0102
3.10	0.3281	0.0015	0.0372	0.3737	0.0419	0.0105
3.00	0.3125	0.0015	0.0401	0.4028	0.0428	0.0107

3.00	0.3125	0.0015	0.0401	0.4028	0.0428	0.0107
2.90	0.3125	0.0015	0.0401	0.4028	0.0443	0.0111
2.80	0.3125	0.0016	0.0401	0.4028	0.0458	0.0115
2.70	0.2969	0.0015	0.0433	0.4349	0.0470	0.0118
2.60	0.2969	0.0016	0.0433	0.4350	0.0488	0.0122
2.50	0.2969	0.0017	0.0433	0.4350	0.0508	0.0127
2.40	0.2891	0.0017	0.0451	0.4524	0.0526	0.0131
2.30	0.2813	0.0017	0.0469	0.4708	0.0546	0.0136
2.20	0.2813	0.0017	0.0469	0.4708	0.0570	0.0142
2.10	0.2734	0.0018	0.0488	0.4902	0.0594	0.0148
2.00	0.2734	0.0018	0.0488	0.4903	0.0624	0.0156
1.90	0.2656	0.0019	0.0509	0.5109	0.0653	0.0163
1.80	0.2656	0.0020	0.0509	0.5110	0.0689	0.0172
1.70	0.2656	0.0021	0.0509	0.5111	0.0730	0.0182
1.60	0.2656	0.0022	0.0509	0.5112	0.0775	0.0193
1.50	0.2656	0.0024	0.0509	0.5114	0.0827	0.0206
1.40	0.2734	0.0026	0.0489	0.4911	0.0890	0.0222
1.30	0.2813	0.0029	0.0470	0.4719	0.0964	0.0240
1.20	0.2969	0.0034	0.0435	0.4365	0.1056	0.0263
1.10	0.3125	0.0040	0.0403	0.4046	0.1165	0.0290

## 17. Computer Program

In the section the FORTRAN program used is listed.

Results of our calculations are given in Ref. 6.

```

C TWO FLUID END GENERATOR
  IMPLICIT REAL*4 (A-H,O-Z)
  COMMON PS(5),TS(5),XN(5),P(6),T(6),RHO(6),S(6),V(4,7),
+  A1A3,A3A4,CP1,CP2,CP3,CM1,CB2,CB3,CF,DELH,DELH13,DELY,
+  DELTS,DELTS4,E,ETAC,ETAC4,ETAX,ETAP,ETAC,FM1,FM2,FM3,FM4,FM5,FMX,
+  FMY,FMAX,F13FMX,F13FMY,GAMMA1,SA44A2,SA44A3,G11,G12,G13,G14,G15,
+  G16,G21,G22,G23,G24,G25,G26,G31,G32,G33,G34,G35,G36,G41,G42,G4X,
+  KAX,KFY,KX0,KX1,FLUID,PVT6,PO,PS1PS7,JI4,K1,R2,R3,
+  S4IX,TJ,WC,WEX,WP,NET,NO,NV,Y,YMIN,YMAX,TZ,X4Z,D1,FMZ,PZ,
+  U(10,12),W(2,3)
C DATA FILE
  U(1,1)=1500.
  U(1,2)=1079.3
  U(1,3)=900.
  U(1,4)=991.7
  U(1,5)=41.4
  U(1,6)=576.8
  U(1,7)=7.17
  U(1,8)=9490.
  U(1,9)=1.
  U(1,10)=18.
  U(1,11)=1.3
  U(1,12)=9490.
  U(2,1)=1500.
  U(2,2)=1089.4
  U(2,3)=300.
  U(2,4)=777.9
  U(2,5)=52.0
  U(2,6)=730.5
  U(2,7)=47.95
  U(2,8)=9490.
  U(2,9)=1.
  U(2,10)=13.
  U(2,11)=1.3
  U(2,12)=9490.
  U(3,1)=1500.
  U(3,2)=1109.2
  U(3,3)=700.
  U(3,4)=962.8
  U(3,5)=64.3
  U(3,6)=738.5
  U(3,7)=43.73
  U(3,8)=9490.
  U(3,9)=1.
  U(3,10)=13.
  U(3,11)=1.3
  U(3,12)=9490.
  U(4,1)=1500.
  U(4,2)=1115.6
  U(4,3)=500.
  U(4,4)=942.9
  U(4,5)=73.7
  U(4,6)=773.3
  U(4,7)=49.75
  U(4,8)=9490.

```



$U(4,9)=1.$   
 $U(4,10)=13.$   
 $U(4,11)=1.3$   
 $U(4,12)=9490.$   
 $U(5,1)=1500.$   
 $U(5,2)=1133.5$   
 $U(5,3)=500.$   
 $U(5,4)=926.7$   
 $U(5,5)=45.7$   
 $U(5,6)=313.9$   
 $U(5,7)=53.76$   
 $U(5,8)=9490.$   
 $U(5,9)=1.$   
 $U(5,10)=18.$   
 $U(5,11)=1.3$   
 $U(5,12)=9490.$   
 $U(6,1)=1500.$   
 $U(6,2)=1945.$   
 $U(6,3)=900.$   
 $U(6,4)=1800.$   
 $U(6,5)=3.$   
 $U(6,6)=126.3$   
 $U(6,7)=740.$   
 $U(6,8)=4000.$   
 $U(6,9)=2.$   
 $U(6,10)=2.016$   
 $U(6,11)=1.4$   
 $U(6,12)=4000.$   
 $U(7,1)=1500.$   
 $U(7,2)=1945.$   
 $U(7,3)=300.$   
 $U(7,4)=1770.$   
 $U(7,5)=11.$   
 $U(7,6)=128.7$   
 $U(7,7)=750.$   
 $U(7,8)=4000.$   
 $U(7,9)=2.$   
 $U(7,10)=2.016$   
 $U(7,11)=1.4$   
 $U(7,12)=4000.$   
 $U(8,1)=1500.$   
 $U(8,2)=1945.$   
 $U(8,3)=700.$   
 $U(8,4)=1730.$   
 $U(8,5)=13.5$   
 $U(8,6)=130.7$   
 $U(8,7)=750.$   
 $U(8,8)=4000.$   
 $U(8,9)=2.$   
 $U(8,10)=2.016$   
 $U(8,11)=1.4$   
 $U(8,12)=4000.$   
 $U(9,1)=1500.$   
 $U(9,2)=2050.$   
 $U(9,3)=600.$

$U(9,4)=1700.$   
 $U(9,5)=17.5$   
 $U(9,6)=135.3$   
 $U(9,7)=750.$   
 $U(9,8)=4000.$   
 $U(9,9)=2.$   
 $U(9,10)=2.015$   
 $U(9,11)=1.4$   
 $U(9,12)=4000.$   
 $U(10,1)=1500.$   
 $U(10,2)=2100.$   
 $U(10,3)=500.$   
 $U(10,4)=1500.$   
 $U(10,5)=19.5$   
 $U(10,6)=137.5$   
 $U(10,7)=750.$   
 $U(10,8)=4000.$   
 $U(10,9)=2.$   
 $U(10,10)=2.016$   
 $U(10,11)=1.4$   
 $U(10,12)=4000.$   
 $V(1,1)=1.$   
 $V(1,2)=1.$   
 $V(1,3)=0.$   
 $V(1,4)=0.$   
 $V(1,5)=1.$   
 $V(1,6)=1.$   
 $V(1,7)=1.$   
 $V(2,1)=.85$   
 $V(2,2)=.85$   
 $V(2,3)=.25$   
 $V(2,4)=.8$   
 $V(2,5)=.93$   
 $V(2,6)=.85$   
 $V(2,7)=1.$   
 $V(3,1)=.7$   
 $V(3,2)=.7$   
 $V(3,3)=1.$   
 $V(3,4)=2.$   
 $V(3,5)=.95$   
 $V(3,6)=.7$   
 $V(3,7)=1.$   
 $V(4,1)=.9$   
 $V(4,2)=1.$   
 $V(4,3)=1.$   
 $V(4,4)=0.$   
 $V(4,5)=1.$   
 $V(4,6)=.9$   
 $V(4,7)=1.$   
 $W(1,1)=.5$   
 $W(1,2)=1.$   
 $W(1,3)=2.$   
 $W(2,1)=.1$   
 $W(2,2)=1.$   
 $W(2,3)=5.$

```

C DATA INPUT
500 WRITE(5,510)
510 FORMAT('0',3X,'ENTER CONTROL CODE')
READ(5,520) IU,IV,IW
520 FORMAT(3I2)
IF(IU.EQ.0) STOP
IF(IU.EQ.-2) GO TO 130
IF(IU.EQ.-3) GO TO 300
IF(IU.EQ.-4) GO TO 232
525 WRITE(5,530)
530 FORMAT('0',3X,'ENTER FLUID PROPERTY CHANGES, IF ANY.')
READ(5,540) JU,ZU
540 FORMAT(12,F9.4)
IF(JU.EQ.0) GO TO 550
IF(JU.EQ.-1) GO TO 500
IF(JU.EQ.-2) GO TO 130
IF(JU.EQ.-3) GO TO 300
IF(JU.EQ.-4) GO TO 232
U(IU,JU)=ZU
GO TO 525
550 WRITE(5,560)
560 FORMAT('0',3X,'ENTER SYSTEM PARAMETER CHANGES, IF ANY.')
READ(5,540) JV,ZV
IF(JV.EQ.0) GO TO 570
IF(JV.EQ.-1) GO TO 500
IF(JV.EQ.-2) GO TO 130
IF(JV.EQ.-3) GO TO 300
IF(JV.EQ.-4) GO TO 232
V(IV,JV)=ZV
GO TO 550
570 WRITE(5,580)
580 FORMAT('0',3X,'ENTER FORMAT CHANGES, IF ANY.')
READ(5,590) JW,ZW
IF(JW.EQ.0) GO TO 130
IF(JW.EQ.-1) GO TO 500
IF(JW.EQ.-2) GO TO 130
IF(JW.EQ.-3) GO TO 300
IF(JW.EQ.-4) GO TO 232
W(IW,JW)=ZW
GO TO 570
130 PS(1)=U(IU,1)
TS(1)=U(IU,2)
P(1)=U(IU,3)
T(1)=U(IU,4)
DELH=U(IU,5)
DELH10=U(IU,6)
PH(6)=U(IU,7)
CB1=U(IU,8)
GAMMA1=1.-ALOG(TS(1)/T(1))/ALOG(PS(1)/P(1))
GAMMA1=1./GAMMA1
G11=(GAMMA1-1.)/2.
G12=G11+1.
G13=GAMMA1/(GAMMA1-1.)
G14=G13-1.
G15=G12/(GAMMA1-1.)

```

```

G10=G10-1.
CPI=DELH/(TS(1)-T(1))
R1=CPI/G10
NG=1.03587/R1
NFLUID=1+IX(0(10,9))
IF(NFLUID).EQ.1) GO TO 400
NS=0(10,10)
GAMMA2=0(10,11)
G32=0(10,12)
GO TO 410
400 G32=CBI
GAMMA2=GAMMA1
NG=NG
410 ETAE=V(IV,1)
ETA0=V(IV,2)
A3A4=V(IV,3)
CF=V(IV,4)
ETAX=V(IV,5)
ETAP=V(IV,6)
PS1PS7=V(IV,7)
DELY=W(IA,1)
YMIN=W(IA,2)
YMAX=W(IA,3)
PJ=14.695
T0=520.
WRITE(6,1+J) PS(1),TS(1),P(1),T(1),DELH,DELH10,RHJ(6),CBI,
+ NFLUID,NG,GAMMA2,G32
1+J FORMAT('J',3X,'FLUID PROPERTIES'//
+ '1 PS1=',F7.1,' PSIA',13X,' 2 TS1=',F7.1,' DEG R'//
+ '3 P1=',F7.1,' PSIA',14X,' 4 T1=',F7.1,' DEG R'//
+ '5 (HS1-H1)=',F7.2,' BTU/LB',5X,' 6 (HS1-H0)=',F7.2,
+ ' BTU/LB'//
+ '7 RHJc=',E12.4,' LB/FT3',5X,' 8 CBI=',E12.4,' M2 DEG K/CM
+ ' '//
+ '9 NFLUID=',12,21X,'10 NG=',E12.4/
+ '11 GAMMA2=',F7.4,10X,'12 G32=',E12.4,' M2 DEG K/CMB')
WRITE(6,150) ETAE,ETA0,A3A4,CF,ETAX,ETAP,PS1PS7
150 FORMAT('J',3X,'SYSTEM PARAMETERS'//
+ '1 ETAE=',F5.2,3X,' 2 ETA0=',F5.2,3X
+ '3 A3/A4=',F5.3,2X,' 4 CF=',F5.2/4X,' 5 ETAX=',F5.2,3X,
+ '6 ETAP=',F5.2,3X,' 7 PS1/PS7=',F5.2)
WRITE(6,155) DELY,YMIN,YMAX
155 FORMAT('J',3X,'COMPUTATIONAL FORMAT'//
+ '1 DELY=',F5.2,3X,' 2 YMIN=',F5.2,3X,' 3 YMAX=',F5.2)
160 Y=YMIN
P(2)=P(1)
P(6)=P(2)
R2=1.03587/NG
T(2)=T(1)
T(6)=T(1)
G21=(GAMMA2-1.)/2.
G22=G21+1.
G23=GAMMA2/(GAMMA2-1.)
G24=G23-1.
G25=G22/(GAMMA2-1.)

```

```

      G26=G25-1.
      X4(1)=(TS(1)/T(1)-1.1/G11
      X4(1)=SQRT(X4(1))
      Y2CR=X4(1)*SQRT(GAMMA1/GAMMA2*R1/R2*T(1)/T(2))
      F41=X4(1)*(1.+G11*X4(1)**2)**(-.515)
      SM1=X4(1)/(1.+GAMMA1*X4(1)**2)*SQRT(1.+G11*X4(1)**2)
      WRITE(6,230) WV, GAMMA1,Y2CR
230  FORMAT('0',4X,'WV=',E12.4,'(EFF MOL WT VAPOR)  GAMMA1=',F7.4,' Y2
      +CR=',F7.4)
      GO TO 500
C  MOMENTUM FUNCTIONS
800  Y=YMAX
810  WRITE(6,320) Y
820  FORMAT('0',4X,'Y=',F7.3)
      X=0.
830  DO 850 L3CP=1,21
      XSMT1=EXCESS(X)
      WRITE(6,340) X,XSMT1
840  FORMAT(9X,'X=',F7.3,' XSMTM=',E12.4)
      X=X+.05
850  CONTINUE
      IF(Y.LE.YMIN) GO TO 500
      Y=Y-DELY
      GO TO 310
C  EQUILIBRIUM AND PERFORMANCE
232  WRITE(6,235)
235  FORMAT('0',3X,'Y',9X,'X',5X,'A1/A3',3X,'OVERALL',
      +2X,'RELATIVE',3X,'B',3X,'M5'
      +37X,'EFF',7X,'EFF')
      Y=YMAX
236  X=XZERO(Y)
300  PS7=PS(1)/P51PS7
      WE=CP3*DELH
      WP=144.7778.26*(PS7-P(6))/RHO(6)
      WEX=WE/X
      IF (WEX.LE.DELH) GO TO 340
      WEX=DELH
      WRITE(6,320)
      FORMAT('0',4X,'AVAILABLE ENTHALPY DROP ATTAINED')
320  WNET=ETAX*WEX-WP/ETAP
340  SIN=DELH16-WP/ETAP
      ETAC=WNET/SIN
      WMAX=ETAX*DELH-WP/ETAP
      ETAR=WNET/WMAX
      WRITE(6,350) Y,X,A1A3,ETAC,ETAR,XM(3),XM(5)
350  FORMAT('0',F9.2,F10.4)
      Y=Y-DELY
      IF(Y.GE.YMIN) GO TO 238
      GO TO 500
      END
C  FUNCTION XMACH
      FUNCTION XMACH(FIN,G)
      IMPLICIT REAL*4(A-F,I,U-Z)
      MFLAG=0
      KOUNT=0

```

```

G1=(G-1.)/2.
G2=G1+1.
G3=G2/(G-1.)
G4=G3-1.
X4N=0.
FMAX=G2**(-G5)
FMNFMX=FMN/FMAX
IF(FMNF4X.LT.1.) GO TO 1000
X4ACH=1.
WRITE(6,1010) FMNF4X
1010 FORMAT('0',3X,'FMN/FMAX=',F7.5,' CHOKING LIMIT EXCEEDED')
RETURN
1000 F=FMN*(1.+G1*X4N**2)**G6-X4N
IF(AMPLAS.EQ.1.) WRITE(6,1030) X4N,F
1030 FORMAT('0',3X,'A=',F7.5,' F=',E12.4)
JELF=ABS(F)
IF(JELF.LT..5E-05) GO TO 1020
F2=FMN*G2*X4N*(1.+G1*X4N**2)**G6-1.
X4N=(X4N-F/F2)
KOUNT=XOUNT+1
IF(KOUNT.LE.20) GO TO 1000
WRITE(6,1040) X4N,F
1040 FORMAT('0',4X,'A=',F7.5,' F=',E12.4,' AFTER 20 ITERATIONS')
1020 X4ACH=X4N
RETURN
END
C FUNCTION XZERO(Y)
FUNCTION XZERO(Y)
IMPLICIT REAL*4 (A-H,O-Z)
ITER=0
XR=1.
E=EXCESS(1.)
ABSE=ABS(E)
IF(ABSE.GE..5E-04) GO TO 700
XZERO=1.
RETURN
700 IF(E.GT.0.) GO TO 720
WRITE(6,710) E
710 FORMAT('0',4X,'X=1.0 E=',E12.4,' MOMENTUM DEFICIT')
XZERO=1.
RETURN
720 XL=0.
E=EXCESS(0.)
ABSE=ABS(E)
IF(ABSE.GE..5E-04) GO TO 730
XZERO=0.
RETURN
730 IF(E.LT.0.) GO TO 750
WRITE(6,740) E
740 FORMAT('0',4X,'X=0.0 E=',E12.4,' MOMENTUM INVERSION')
XZERO=0.
RETURN
750 X=(XL+XR)/2.
E=EXCESS(X)
ABSE=ABS(E)

```

```

ITER=ITER+1
IF (ITER.LE.13) GO TO 780
WRITE(6,700)X,E
790 FORMAT('0',4X,'X=',F7.5,' E=',E12.4,' AFTER 13 ITERATIONS')
XZERO=X
RETURN
780 IF (ABS(GE..5E-04) GO TO 760
XZERO=X
RETURN
760 IF (E.LT.0.) GO TO 770
XR=X
GO TO 750
770 XL=X
GO TO 750
END
C FUNCTION EXCESS(X)
FUNCTION EXCESS(X)
IMPLICIT REAL * 4 (A-H,O-Z)
COMMON PS(5),TS(5),X4(5),P(5),RHO(5),S(5),V(4,7),
+ A1A3,A3A4,CP1,CP2,CP3,CB1,CB2,CB3,CF,DELH,DELH16,DELY,
+ UELTS,VELTS+,E,ETA,ETAE,ETAO,ETAX,ETAP,ETAC,FM1,FM2,FM3,FM4,FM5,FMX,
+ FMY,FMAX,FM3FMY,FMYFMA,GAMMA1,GAMMA2,GAMMA3,G11,G12,G13,G14,G15,
+ G16,G21,G22,G23,G24,G25,G26,G31,G32,G33,G34,G35,G36,G37,G38,G39,G40,G41,G42,G43,
+ GAMX,XMY,XSU,AK1,DELTO,PVT6,PD,PS1,PS7,DIN,R1,R2,R3,
+ S41X,T0,WX,WEX,WP,WNET,WG,WV,Y,YMIN,YMAX,TZ,XM2,D1,FMZ,PZ,
+ J(10,12),N(2,3)
XM(2)=(X4(1)/Y)*SQRT((GAMMA1/GAMMA2)*(R1/R2)*(T(1)/T(2)))
IF (XM(2).LT.1.) GO TO 239
XM(2)=1.
T(2)=T(1)/Y**2*(GAMMA1/GAMMA2)*R1/R2
239 TS2T2=1.+G21*XM(2)**2
TS(2)=T(2)*TS2T2
PS(2)=P(2)*TS2T2**G23
G42=XM(2)/(1.+GAMMA2*XM(2)**2)*SQRT(1.+G21*XM(2)**2)
240 R3=X*F1+(1.-X)*R2
CP2=G23*R2
CP3=X*CP1+(1.-X)*CP2
GAMMA3=CP3/(CP3-R3)
G31=(GAMMA3-1.)/2.
G32=G31+1.
G33=GAMMA3/(GAMMA3-1.)
G34=G33-1.
G35=G32/(GAMMA3-1.)
G36=G35-1.
FMAX=G22**(-G35)
TS(3)=(X*CP1*TS(1)+(1.-X)*CP2*TS(2))/CP3
FM2=XM(2)*(1.+G21*XM(2)**2)**(-G25)
D1=PS(2)/PS(1)*FMY2/FM1*SQRT(R1/R2*TS(1)/TS(2)*GAMMA2/GAMMA1)
A1A3=X*D1/(1.-X+X*D1)
S(1)=CP1*ALOG(TS(1)/T0)-R1*ALOG(PS(1)/P0)
S(2)=CP2*ALOG(TS(2)/T0)-R2*ALOG(PS(2)/P0)
PSX=(CP3*ALOG(TS(3)/T0)-X*S(1)-(1.-X)*S(2))/R3
PSX=PO*EXP(PSX)
D2=PS(2)/PSX*FM2*SQRT(R3/R2*TS(3)/TS(2)*GAMMA2/GAMMA3)
FMX=D2/(1.-X+X*D1)

```

```

IF(FMX.LE.FMAX) GO TO 241
XMX=1.
FMXFM=FMX/FMAX
FMX=FMAX
WRITE (6,243) Y,X,FMXFMX
243 FORMAT('J',4X,'Y=',F7.5,' X=',F7.5,' FMX/FMAX=',E12.4)
GO TO 244
241 XMX=XMACH(FMX,GAMMA3)
244 GMY=SQRT((3*TS(3)/GAMMA3)/(X/G41*SQRT(R1*TS(1)/GAMMA1)+(1.-X)/GM2*
+SQRT(R2*TS(2)/GAMMA2)))
GMAX=1./SQRT(2.*(GAMMA3+1.))
IF(GMY.LE.GMAX) GO TO 245
GMYGMX=GMY/GMAX
WRITE (6,242) Y,X,GMYGMX
242 FORMAT('J',4X,'Y=',F7.5,' X=',F7.5,' GMY/GMAX=',E12.4/
+BX,'MOMENTUM LIMIT EXCEEDED.')
XMY=1.
FMY=FMAX
GO TO 248
245 XMY=((1.-2.*(GAMMA3*GMY**2)-SQRT(1.-2.*(GAMMA3+1.)*GMY**2)))/
+(1.-GAMMA3*(1.-2.*(GAMMA3*GMY**2)))
XMY=SQRT(XMY)
FMY=XMY/(1.+G31*XMY**2)**G35
IF(FMY.LE.FMAX) GO TO 248
XMY=1.
FMYFMAX=FMY/FMAX
FMY=FMAX
WRITE (6,246) Y,X,FMYFMAX
246 FORMAT('J',4X,'Y=',F7.5,' X=',F7.5,' FMY/FMAX=',E12.4)
243 PSY=PSX*FMAX/FMY
PS(3)=PSX*((PSY/PSX)**(1./ETA1))
FM3=PSX/PS(3)*FMAX
IF(FM3.LE.FMAX) GO TO 249
X4(3)=1.
WRITE(6,900) X4(1),X4(2),X4X,XMY,X4(3),FM1,FM2,FMX,FMY,FM3,
+FM1X,G41,G42,GMY,GMAX,PS(1),PS(2),PSX,PSY,PS(3),TS(1),
+TS(2),TS(3)
900 FORMAT('J',6X,'M1,12,MX,MY,M3=1/5E12.4/
+'0',F41,F42,F4X,F4Y,F43,F4MAX=1/5E12.4,E11.4/
+'0',G41,G42,G4Y,G4MAX=1/4E12.4/
+'0',PS1,PS2,PSX,PSY,PS3=1/5E12.4/
+'0',TS1,TS2,TS3=1/5E12.4)
FM3FMX=FM3/FMAX
FM3=FMAX
WRITE (6,251) Y,X,FM3FMX
251 FORMAT('J',4X,'Y=',F7.5,' X=',F7.5,' FM3/FMAX=',E12.4)
GO TO 247
249 X4(3)=X4ACH(FM3,GAMMA3)
247 TS3T3=1.+G31*X4(3)**2
T(3)=TS(3)/TS3T3
PS3P3=TS3T3**G33
P(3)=PS(3)/PS3P3
PS(4)=PS(3)*EXP(-(1.-ETA0)*GAMMA3/2.*X4(3)**2)
FM4=PS(3)/PS(4)*A314*FM3
IF(FM4.LE.FMAX) GO TO 252

```



```

X4(4)=1.
FM4FMX=FM4/FMAX
FM4=FMX
WRITE(3,253) Y,X,FM4FMX
253 FORMAT('J',+X,'Y=',F7.5,' X=',F7.5,' FM4/FMAX=',E12.4)
GO TO 254
252 X4(4)=X4ACH(FM4,GAMMA3)
25+ TS(4)=TS(3)
TS4T4=1.+G31*X4(4)**2
T(+)=TS(4)/TS4T4
PS4P4=TS4T+*G33
P(4)=PS(4)/PS4P4
C33=X*C31+(1.-X)*C32
BETA=C.000075*(P0/14.696)*(C83/)*90.*520./T0)**2
DELT54=BETA*G31/GA4MA3*PS(4)/P0*(T0/TS(4))**2/(1.+G31*X4(4)**2)**
+G34
DELT5=TS(4)*DELT54
TS(5)=TS(4)-DELT5
PS(5)=PS(4)*(TS(5)/TS(4))*G33
FM5=TS(5)/TS(4)*PS(4)/PS(5)*FM4
IF(FM5.LE.FMAX) GO TO 250
X4(5)=1.
FM5FMX=FM5/FMAX
FM5=FMX
WRITE(3,255) Y,X,FM5FMX
255 FORMAT('J',+X,'Y=',F7.5,' X=',F7.5,' FM5/FMAX=',E12.4)
GO TO 260
250 X4(5)=X4ACH(FM5,GAMMA3)
260 TS5T5=1.+G31*X4(5)**2
T(5)=TS(5)/TS5T5
PS5P5=TS5T5*G33
P(5)=PS(5)/PS5P5
FM2=(1.-X)*A3A4*FM2/(1.-X+X*D1)
AM2=XACH(FM2,GAMMA2)
TSZT7=1.+G21*AM2**2
TZ=TS(2)/TSZT7
PZ=PS(2)/(TSZT7*G23)
EXCESS=1.-PZ/P(5)+((1.-X-CF/2.)*GA4MA3*X4(5)**2-(1.-X)*GAMMA3*
+X4(5)*AM2*SQRT(GAMMA2/GAMMA3*R2/R3*TZ/T(5)))
RETURN
END

```

## 18. Nomenclature

$A$	- area
$a$	- speed of sound
$C_B$	- characteristic breakdown constant (Eqn 10.3)
$C_p$	- specific heat at constant pressure
$c_f$	- friction loss coefficient - condenser
$E_b$	- breakdown electric field strength
$E(x)$	- Eqn. 11.5
$F(M_n)$	- Eqn. 5.16
$f(M)$	- Eqn. 5.3
$f_{max}$	- Eqn. 5.15
$g(M)$	- Eqn. 7.5
$g$	- Eqn. 7.7
$g_{max}$	- Eqn. 7.9
$g_{\infty}$	- Eqn. 7.10
$h$	- enthalpy
$I$	- irreversibility
$M$	- Mach number
$\dot{m}$	- mass flow rate
$p_e$	- gross electrical power
$p$	- pressure
$q_{in}$	- heat input
$R$	- gas constant
$\bar{R}$	- universal gas constant
$s$	- entropy
$s_m$	- entropy of mixing

T	-	temperature
V	-	velocity
W	-	molecular weight
$w_e^*$	-	gross work out per unit mass
$w_p^*$	-	ideal pump work per unit mass
$w_{net}$	-	useful electrical output per unit mass
x	-	$\dot{m}_1/\dot{m}_3$
y	-	$v_1/v_3$
z	-	quality of vapor
$\beta$	-	Eqn. 10.7, dimensionless breakdown constant
$\gamma$	-	ratio of specific heats
$\Delta( )$	-	change of
$\epsilon$	-	permittivity of the medium
$\eta_c$	-	overall cycle efficiency
$\eta_E$	-	Eqn. 8.13, ejector effectiveness
$\eta_D$	-	Eqn. 9.6, diffuser effectiveness
$\eta_p$	-	pump efficiency
$\eta_x$	-	excitation efficiency
$\rho$	-	gas density
$\psi$	-	steady flow availability
$\omega$	-	mass ratio of condensible primary to dry secondary

### Subscripts

- 0 - ambient conditions
- 1 - primary, entrance to ejector (exit from boiler/superheater)
- 2 - secondary, entrance to ejector (exit from separator)
- 3 - mixture, exit from ejector entrance to diffuser
- 4 - mixture, exit from diffuser entrance to generator
- 5 - mixture, exit from generator entrance to condenser
- 6 - primary, exit from condenser entrance to pump
- 7 - primary, exit from pump entrance to boiler
- f - saturated liquid
- G - gas or secondary
- g - saturated vapor
- n - next value
- s - stagnation
- v - vapor
- x - exit station of hypothetical minimum loss device
- y - exit station of ideal ejector
- z - exit station of condenser

## 19. References

1. T. H. Gawain and O. Biblarz, "Performance Analysis of a Type of Electrohydrodynamic Power Generator," NPS67-79-006 (April 1979).
2. T. H. Gawain and O. Biblarz, "Performance Analysis of a Type of Electrohydrodynamic Generator," Proceedings of the 14th IECEC (Boston, August 1979), pp. 162-167.
3. T. H. Gawain and O. Biblarz, "Ejector-Augmented Electrohydrodynamic Power Generator," to appear in Advanced Energy Concepts, C. Solbrig (ed.), Hemisphere Press (1981).
4. N. M. Huberman, H. Shelton, W. Krieve, and C. L. Dailey, "Study of Electrofluid Dynamic Power Generation," AFAPL-TR-76-31 (July 1976).
5. T. H. Gawain and O. Biblarz, "Analysis of a Two-Fluid EHD Power Generator Including the Effects of Compressibility," NPS67-80-002 (January 1980).
6. T. H. Gawain and O. Biblarz, "Estimated Performance of an Electrohydrodynamic Power Generator which Utilizes a Two-Fluid Ejector," preprint AIAA-80-1341 (Snowmass Co., 1980).
7. G. B. Wallis, One-Dimensional Two-Phase Flow, McGraw-Hill, N. Y. (1969).
8. S. H. Hasinger, "Comparison of Experiment and Analysis for a High Primary Mach Number Ejector," AFFDL-TR-23 (June 1978).
9. B. H. Anderson, "Factors which Influence the Analysis and Design of Ejector Nozzles," AIAA Paper No. 72-46 (1972).
10. D. E. Groesbeck, R. G. Huft, and U. H. von Glahn, "Comparison of Jet Mach Number Decay Data with Correlation and Jet Spreading Contours for a Large Variety of Nozzles", NASA TN D-8423 (1977).

DISTRIBUTION LIST

	No. of Copies
1. Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
2. Library Code 0142 Naval Postgraduate School Monterey, CA 93940	2
3. Dean of Research Code 012 Naval Postgraduate School Monterey, CA 93940	1
4. Chairman Department of Aeronautics Code 67 Naval Postgraduate School Monterey, CA 93940	1
5. Dr. Ryszard Gajewski, Director Division of Advanced Energy Projects Department of Energy Mail Stop G-256, Germantown Washington, DC 20545	2
6. Mr. Alvin Marks, Chief Scientist Marks Polarized Corporation 153-16 10th Avenue Whitestone, NY 11357	1
7. Professor T. H. Gawain Department of Aeronautics Code 67Gn Naval Postgraduate School Monterey, CA 93940	3
8. Professor O. Biblarz Department of Aeronautics Code 67Bi Naval Postgraduate School Monterey, CA 93940	5

	No. of Copies
9. Dr. H. R. Rosenwasser Naval Air Systems Command Code AIR 310C Washington, D. C. 20360	1
10. Dr. S. A. Satkowski Office of Naval Research Power Program, Code 473 Washington, D. C. 20360	1
11. Dr. S. Hasinger Thermomechanics Branch AFFDL Wright-Patterson AFB, Ohio 45433	1
12. Dr. C. D. Hendricks Lawrence Livermore Laboratory L-482 P. O. Box 808 Livermore, CA 94550	1
13. Dr. H. Velkoff Department of Mechanical Engineering Ohio State University Columbus, Ohio 43210	1
15. Dr. Hans von Ohain c/o University of Dayton Research Institute Dayton, Ohio 45469	1
16. Dr. J. R. Melcher Department of Electrical Engineering Room 36-319 Massachusetts Institute of Technology Cambridge, MA 02139	1
17. Dr. J. E. Minardi University of Dayton Research Institute Dayton, Ohio 45409	1
18. Mr. M. O. Lawson University of Dayton Research Institute Dayton, Ohio 45409	1
19. Dr. James Reilly W. T. Schafer Associates 10 Lakeside Office Park Wakefield, MA 01880	1